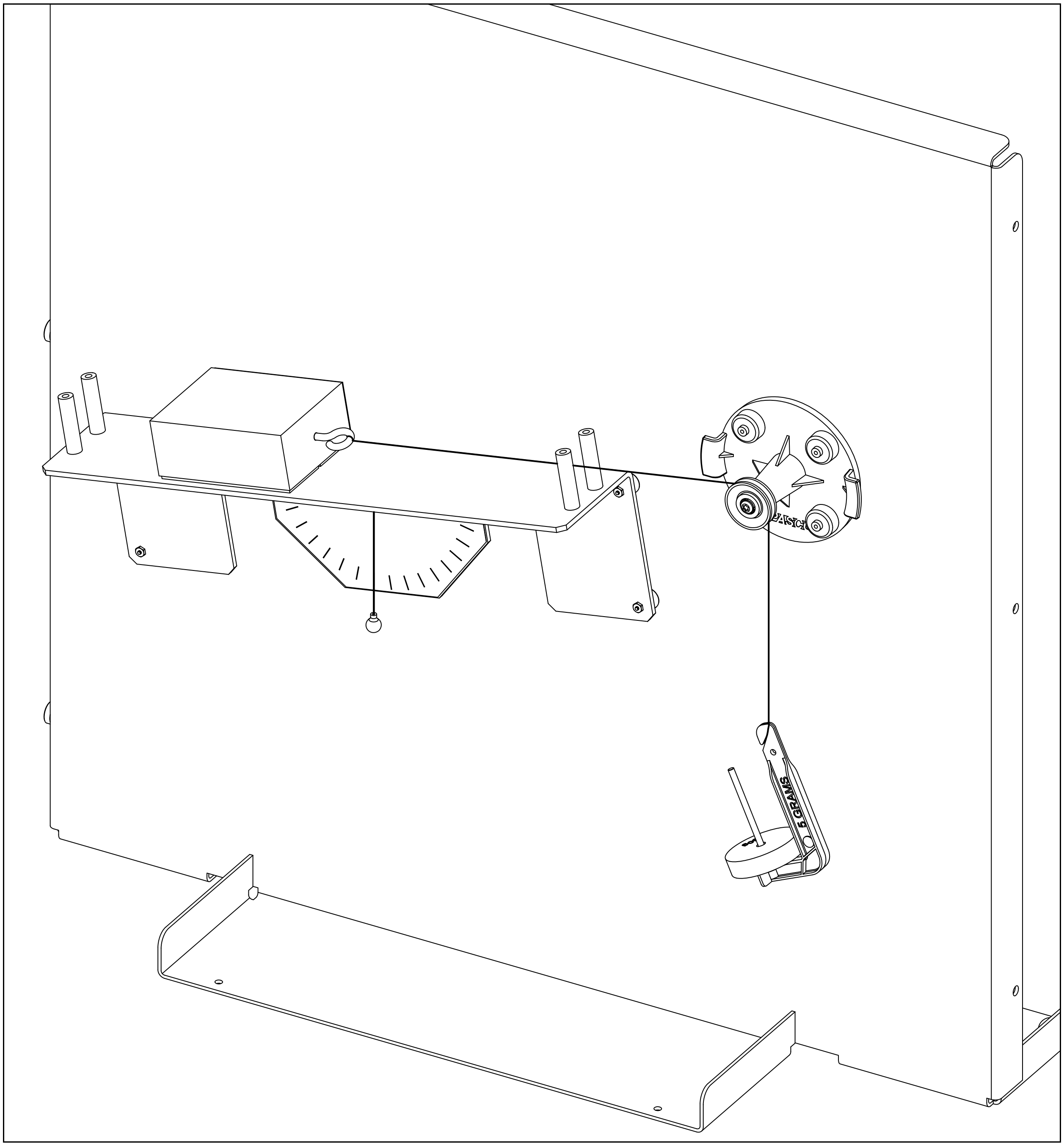
Instruction Manual
012-12876B

PASCO Mechanics

Statics System

ME-9502



The cover page shows a a Friction Block on the Statics System Inclined Plane with a PASCO Mass Hanger from the ME-8979 Mass and Hanger Set suspended by a thread over a Small Pulley. Most of the components of the Statics System are held magnetically to the included workboard. This manual contains fifteen copy-ready experiments about the fundamentals of statics.

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Statics System

ME-9502

Introduction

The study of mechanics often begins with Newton's Laws of Motion. The first law describes the conditions for an object to maintain its state of motion. If the net force on an object is zero, the acceleration of the object is zero.

If $\Sigma F = 0$, then a = 0

The second law describes what happens if the net force on an object is *not* zero. The acceleration is directly proportional to net force, in the same direction as the net force, and inversely proportional to mass, or

$$a = \frac{F_{net}}{m}$$

Much of what is studied in an introductory course deals with the ways that forces interact with physical bodies. The PASCO Statics System is designed to help you investigate the nature of forces for the special case in which there is no acceleration. In other words, the vector sum of all the forces acting on the body is zero.

One reason to study the case of no acceleration is because it is easier to measure non-accelerating systems than it is to measure accelerating systems. A great deal can be learned about the vector nature of forces by studying the many ways in which forces can be applied to an object without causing acceleration.The second reason is that in our everyday experience, non-accelerated systems are the rule, not the exception.

Equipment

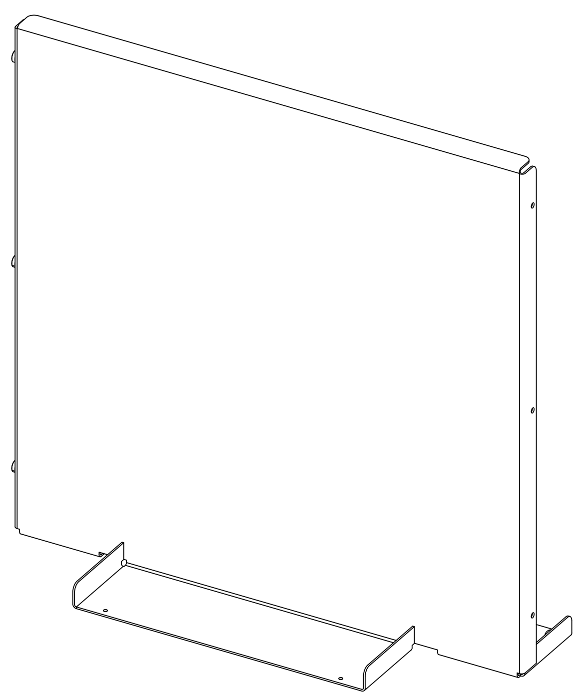
The ME-9502 Statics System consists of the Statics Board, Spares Package, Components Package, and Mass and Hanger Set.

ME-9503 Statics Board

The Statics Board is a ferrous metal plate approximately 45 by 45 centimeters (18 by 18 inches) with a writable white board finish on both sides.)

The board has rubber bumpers on its base. White board pens (or "dry erase" pens) can be used to write and draw directly on the board. The pens are available from stationery stores and the ink can be erased with a cloth or a white board eraser.

The Mounted Scale Assembly, Large and Small Pulley Assemblies, Balance Arm Assembly, Force Wheel Assembly, Torque Wheel Assembly, Inclined Plane Assembly, and Utility Mount Assembly all attach magnetically to the board. Components can be stored on the rear side of the board when not in use. Except for the Inclined Plane Assembly, all the components that can mount on the board have a rubber ring on the base that protects the board and the component. The round magnetic base is 6.5 centimeters (cm) in diameter.

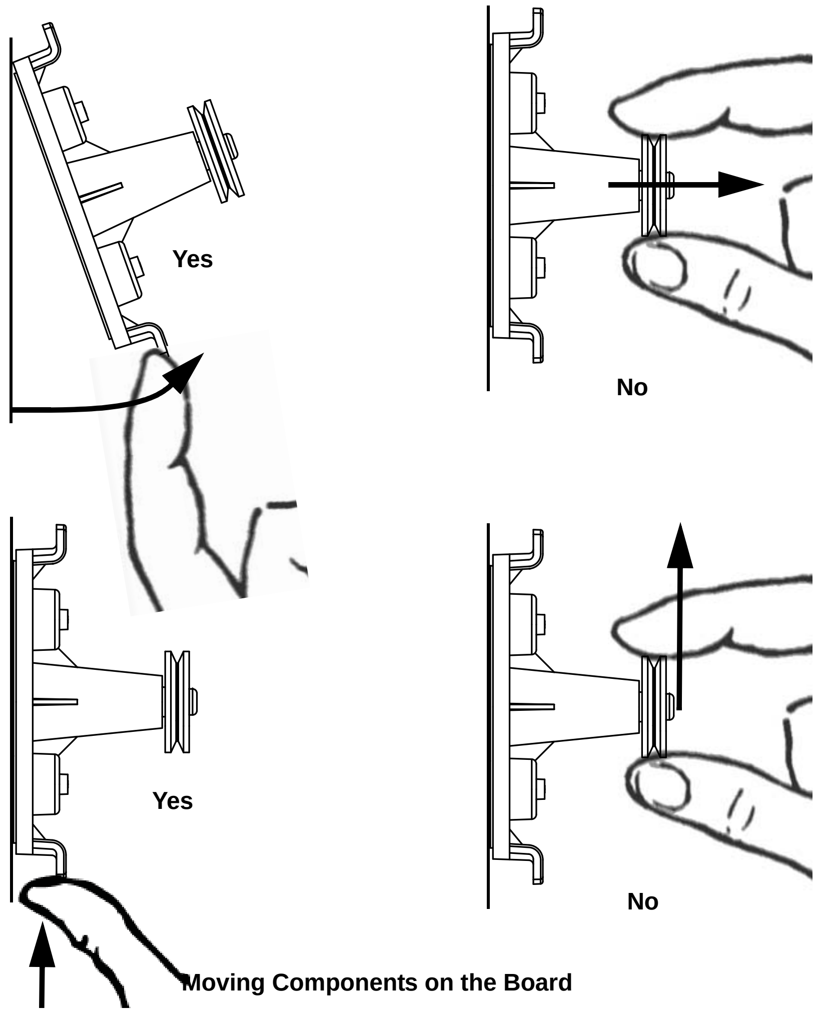


Statics Board

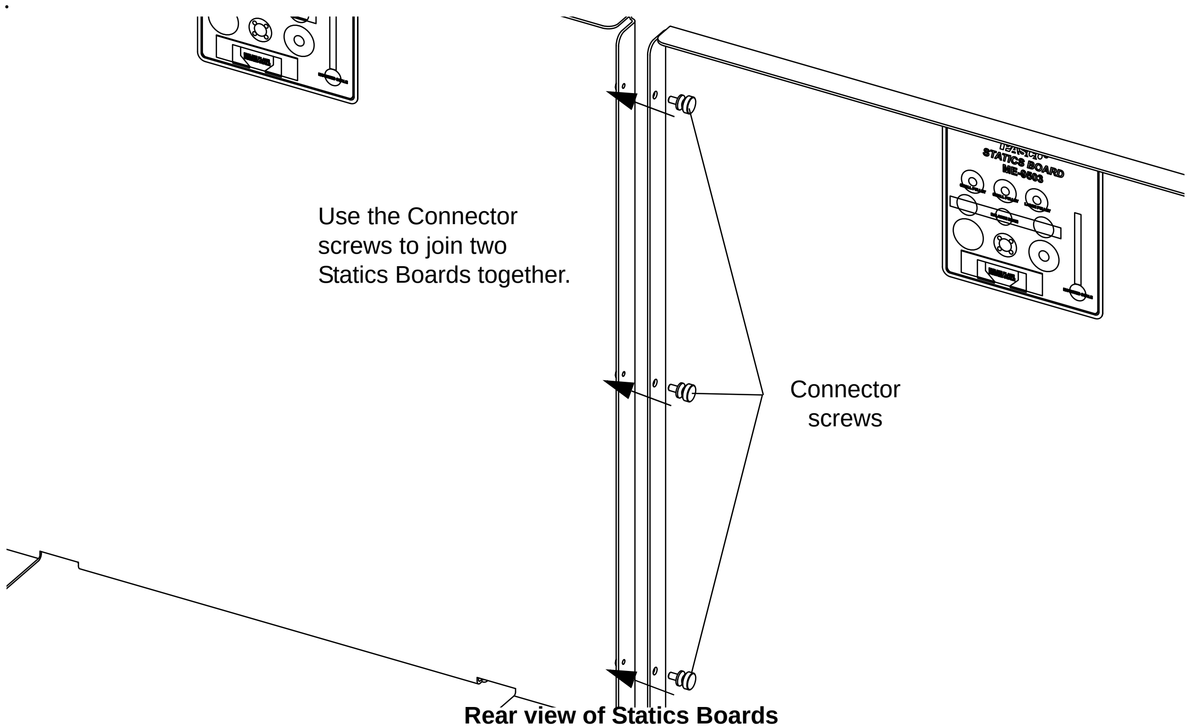
Important

When moving or removing any of the magnetically mounted components from the board, handle the component by the magnetic base rather than by the component itself. This will reduce the strain on the component.

A second board (available separately) can be attached to the first board with the three included connector screws. To join the boards, remove the connector screws from the threaded holes on the side of the first board. Put the edge of the second board next to the first board. Align the holes and join the two boards with the connector screws



Moving Components on the Board



Rear view of Statics Boards

ME-9504 Spares Package

Item	Qty	Item	Qty
String Tie Assembly	3	Thumbscrew 6-32 by 5/8” (for Cord Clip)	2
Torque Indicator Assembly	6	Thumbscrew 4-40 by 1/2” (for balance arm protractor)	2
Nylon Thread, spool	1	Thumbscrew 6-32 by 1/4” (for balance arm pivot)	1
Cord Tensioning Clip	2	Washer 0.285” OD (for balance arm protractor)	2
Angle Indicator Arm	2	Plumb bob, brass (for inclined plane)	1

ME-9505 Components Package

Item	Qty	Item	Qty
Large Pulley Assembly	1	Utility Mount Assembly	1
Small Pulley Assembly	2	Mass Cart Assembly	1
Mounted Scale Assembly	1	Torque Wheel Assembly	1
Balance Arm Assembly	1	Inclined Plane Assembly	1
Force Wheel Assembly	1	Friction Block Assembly	1
Double Pulley Assembly	1	Asymmetrical Plate	1

ME-8979 Mass and Hanger Set

Components from the ME-8979 PASCO Mass and Hanger Set are used in most of the statics experiments. The set includes four mass hangers, a storage box, and twenty-seven masses ranging from 0.5 g to 100 g made from brass, aluminum, or polycarbonate plastic.

ME-8979 Mass and Hanger Set

Recommended Equipment*

Load Cell and Load Cell Amplifier

A PS-2201 5 N Load Cell can be used to measure force. When it is connected through an amplifier to a PASCO interface or to a PASCO hand-held data logger, the force data from the Load Cell can be recorded, displayed, and analyzed.

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PS-2201 5 N Load Cell

Amplifier	Model
Load Cell Amplifier (PASPORT)	PS-2198
Dual Load Cell Amplifier	PS-2205
Load Cell Amplifier (ScienceWorkshop)	CI-6464

*See the PASCO catalog or Web site at **www.pasco.com** for information about PASCO load cells, load cell amplifiers, interfaces, hand-held data loggers, and data acquisition software.

Stopwatch (ME-1234)

The PASCO Stopwatch has a liquid crystal display (LCD) with two display modes. Its precision is 0.01 seconds up to 3599.99 seconds, and 1 second up to 359999 seconds. Its memory holds up to nine event times.

Stop Watch

Mass Balance

Use a mass balance such as the OHAUS Scout Pro Balance 2000 g (SE-8757A) or the OHAUS Cent-O-Gram Balance (SE-8725).

Mass Balances

Mounted Scale Assembly (ME-9824A)

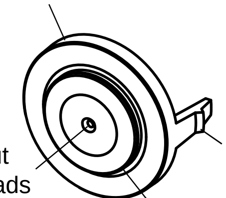
The Mounted Scale Assembly (included in the ME-9505 Components Package) is also available separately as a replacement or as an extra spring scale for experiments where measuring more than one force is required.

Spares Package (ME-9504)

The Spares Package contains some items which are used with assemblies from the Components Package. For example, the String Tie Assembly snaps into the center of the Force Wheel Assembly and the Torque Indicator Assembly snaps into the Torque Wheel Assembly. The Cord Tensioning Clip is used on the Utility Mount Assembly. Other items such as the Angle Indicator Arms, thumbscrews, washers, and brass weight are replacement parts.

String Tie Assembly.

When the String Tie Assembly is snapped into the center of the Force Wheel Assembly, you can use the apparatus to study force vectors. The String Tie Assembly has two parts: a clear plastic inner part (the "force disk") that is free to move around in the center of the outer part, which has a center hole and two tabs that snap into the Force Wheel Assembly. (The two parts are not meant to be separated.)



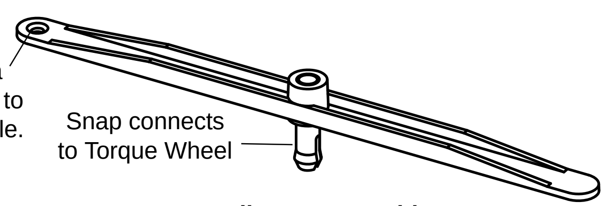
String Tie Assembly

Initial Setup

Get three threads that are 38 cm (15") long. Put the threads through the hole in the inner part of the String Tie Assembly. Tie the ends of the threads together (for example, with an overhand knot) so that they cannot be pulled back through the inner part.

Torque Indicator Assembly

Note that the Torque Indicator Assembly has a snap that can be connected to the Torque Wheel Assembly. The other three assemblies are spares.



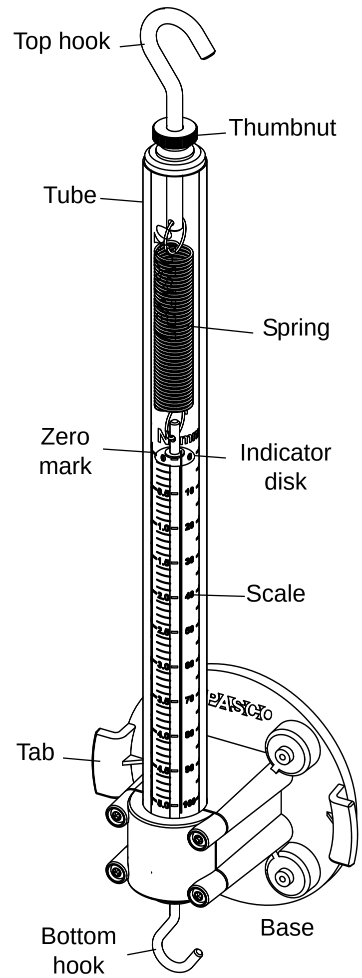
Torque Indicator Assembly

Initial Setup

Tie a 30 cm (12") thread to each of three Torque Indicator Assemblies.

Components Package (ME-9505)

Mounted Scale Assembly



Mounted Scale Assembly

The mounted scale has four strong magnets in its base. The tube is marked in newtons (N), ounces (oz.) and millimeters (mm). The thumbnut allows the top hook to be raised or lowered in order to align the red indicator disk with the zero mark on the tube.

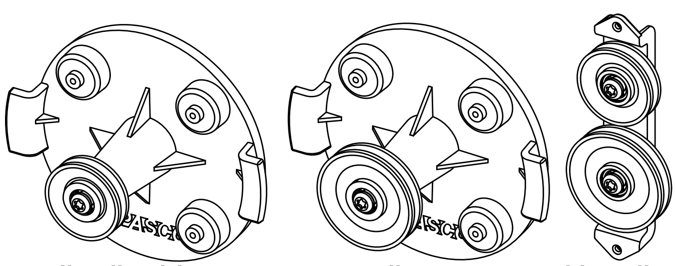
To zero the scale, mount the scale on the statics board. Leave the bottom hook empty. Unscrew the thumbnut a few turns. Rotate the top hook clockwise (left-to-right) to lower the indicator disk and counter-clockwise (right-to-left) to raise the indicator disk. When the disk is aligned with the zero mark on the tube label, tighten the thumbnut to hold the top hook in place.

To move the scale on the statics board, hold the scale by the tabs on the base and push or pull the scale to the desired location. To remove the scale from the statics board, lift one or both of the tabs on either side of the base away from the board.

Large and Small Pulleys and Double Pulley Block

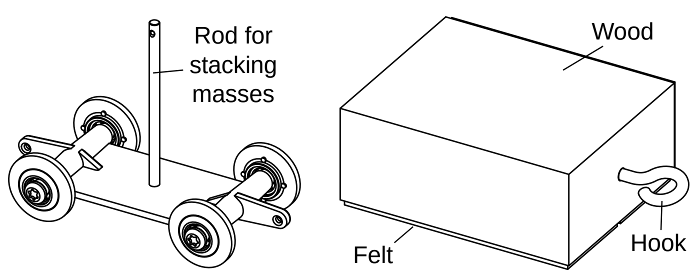
The Statics System comes with two Small Pulleys and one Large Pulley mounted on magnetic bases. It also includes a Double Pulley Block (that does not have a magnetic base). The Double Pulley Block is designed for block-and-tackle experiments. A support thread can be tied to either end of the Double Pulley Block.

All the pulleys have low friction ball bearings. The smaller pulleys are 2.41 cm (0.850”) outside diameter (OD) and 1.65 cm (0.650”) inside diameter (ID). The larger pulleys are 2.79 cm (1.10”) OD and 2.28 cm (0.90”) ID.


Small Pulley (2) **Large Pulley** **Double Pulley Block**

Please use the tabs on the base of the components when moving them around or removing them from the statics board.

Mass Cart and Friction Block



Mass Cart **Friction Block**

The Mass Cart and Friction Block are designed to be used with the Inclined Plane Assembly. The Mass Cart has low friction ball bearings in each wheel and a metal rod in the center for stacking extra masses. A thread can be attached at either end.

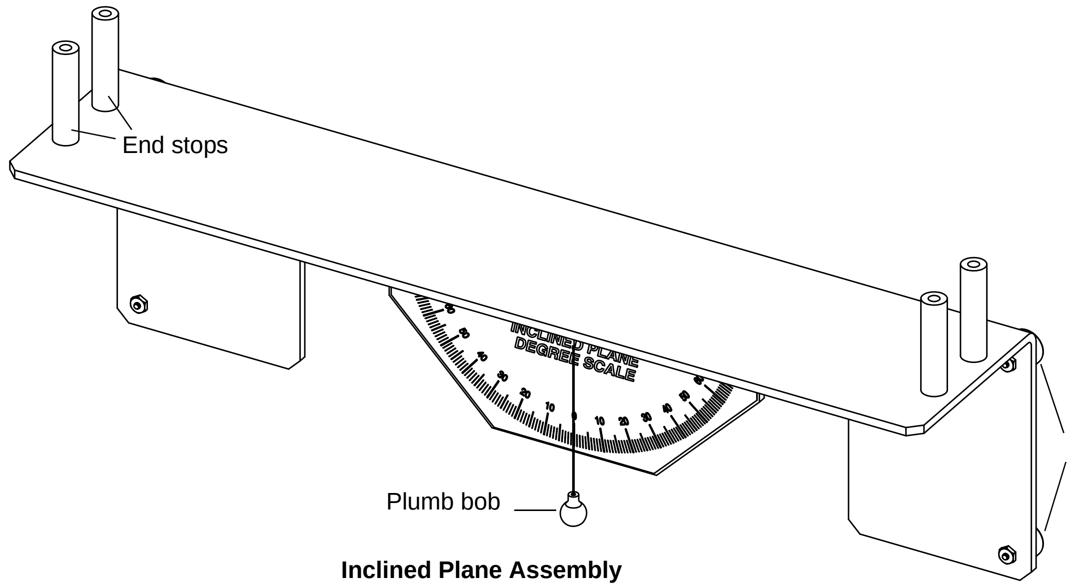
The Friction Block is made of beech wood. It has an “eye” hook at one end and is covered with felt on the bottom and one side. Its dimensions are 2.54 cm by 5.08 cm by 6.35 cm (1.0” by 2.0” by 2.5”).

Inclined Plane Assembly

The Inclined Plane Assembly has four strong magnets on its back side that hold it in position on the Statics Board.

CAUTION: The magnets on the Inclined Plane Assembly are not covered with protective material. Be careful to firmly hold the inclined plane when placing it on the board so that the magnets are not damaged by "snapping" against the board.

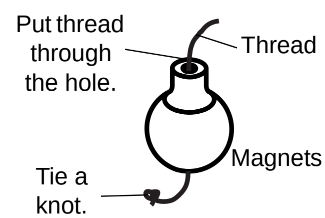
The inclined plane has end stops at each end of the plane and a degree scale with a brass plumb bob for determining the angle of the incline.



Inclined Plane Assembly

Replacing the Plumb Bob

The ME-9504 Spares package includes a spare plumb bob (brass weight). To replace the plumb bob, use a Phillips head ("crosshead") screwdriver to loosen the screw on the back of the degree scale. Remove the piece of thread and get a new piece approximately 10 cm (4") long. Put one end of the thread through the hole in the brass weight and tie a double knot in that end of the thread so it will not slip back through the hole. Put the other end of the thread through the small hole near the top of the degree scale. Adjust the length of the thread so the plumb bob can swing freely and then wind the end of the thread around the screw on the back. Tighten the screw to hold the thread in place.

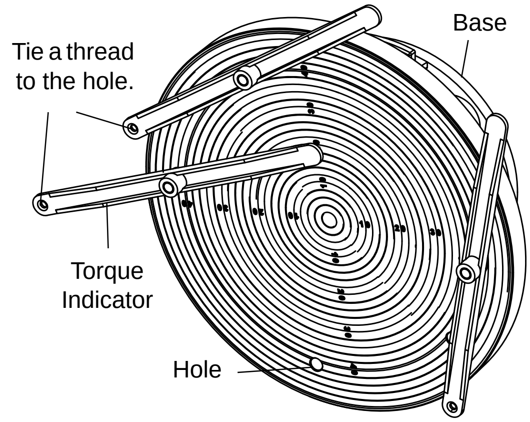


Brass Weight

Torque Wheel Assembly and Torque Indicator Assembly

The Torque Wheel Assembly has a 5 cm radius disk with a label that has concentric circles 2 mm apart. The disk rotates freely on a ball bearing and has five holes for attaching a Torque Indicator Assembly. The Torque Indicator snaps into the hole and rotates freely. Tie a 30 cm (12”) thread to the hole in the end of each indicator.

To remove a Torque Indicator, firmly grasp the indicator at the pivot in the middle and pull outwards while pushing on the end of the snap from the other side of the disk.



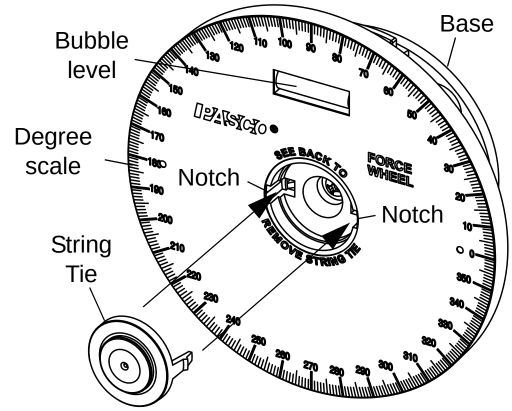
Torque Wheel Assembly
Torque Indicator Assembly

Force Wheel Assembly and String Tie Assembly

The Force Wheel Assembly has a 5 cm radius disk with a 360° label and a bubble level for leveling the Force Wheel. Unlike the Torque Wheel, the Force Wheel does not rotate freely but the disk can be turned in order to level the wheel. Once leveled, the wheel will stay in position until moved again.

REMINDER: Be sure to put tie three threads through the hole in the center of the String Tie before putting the String Tie into the Force Wheel.

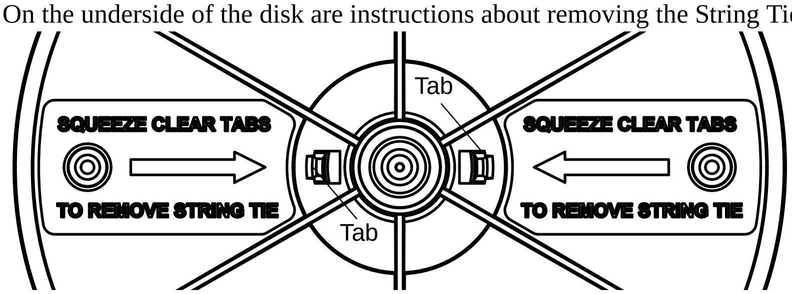
The String Tie Assembly snaps into the hole in the center of the Force Wheel Assembly. Align the tabs on the String Tie with the notches inside the hole at the center of the Force Wheel. Push the String Tie firmly into the Force Wheel until both tabs “snap” into place. Note that the surface of the outer part of the String Tie will be very slightly below the surface of the Force Wheel, but the inner part of the String Tie will be flush with the Force Wheel.



Force Wheel Assembly
String Tie Assembly

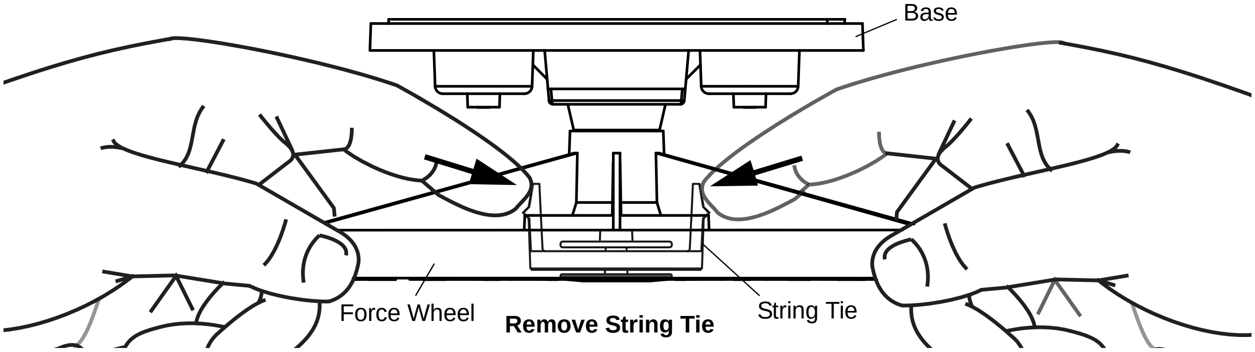
Removing the String Tie Assembly

If you need to remove the String Tie (in order to replace the threads, for example), remove the Force Wheel from the Statics Board. On the underside of the disk are instructions about removing the String Tie.



Remove String Tie

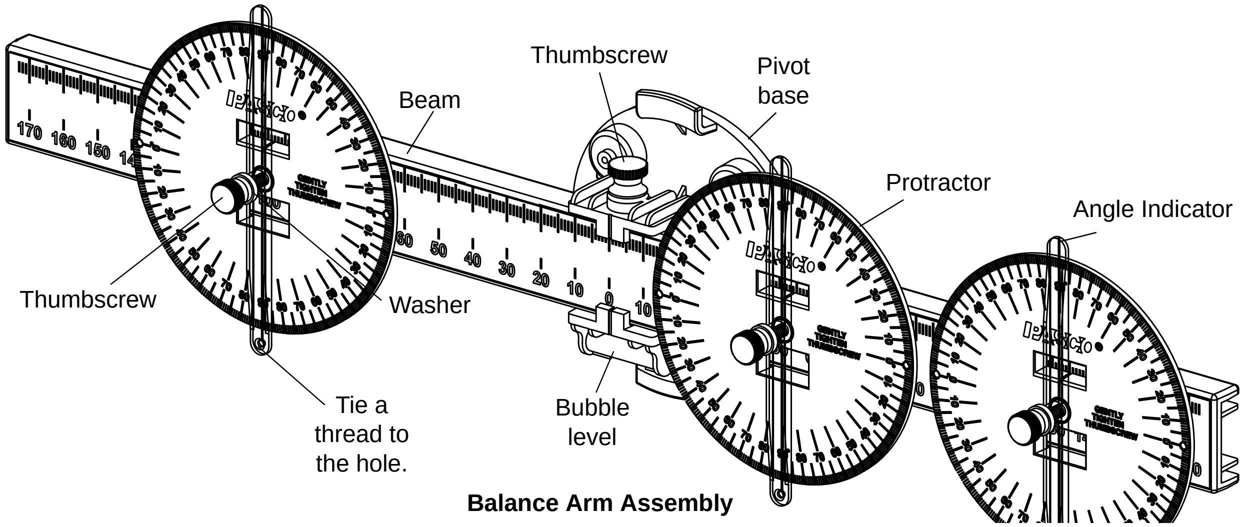
(Imagine that the Force Wheel disk is *transparent* so that you can see the String Tie.) Hold the Force Wheel as shown and push the clear plastic tabs inward with your forefingers until the String Tie pops out of the Force Wheel.



Remove String Tie

Balance Arm Assembly

The Balance Arm Assembly consists of a Beam, a Pivot, and three Protractors that can be mounted on the beam. Each protractor has a transparent Angle Indicator. The pivot has a bubble level for leveling the beam.

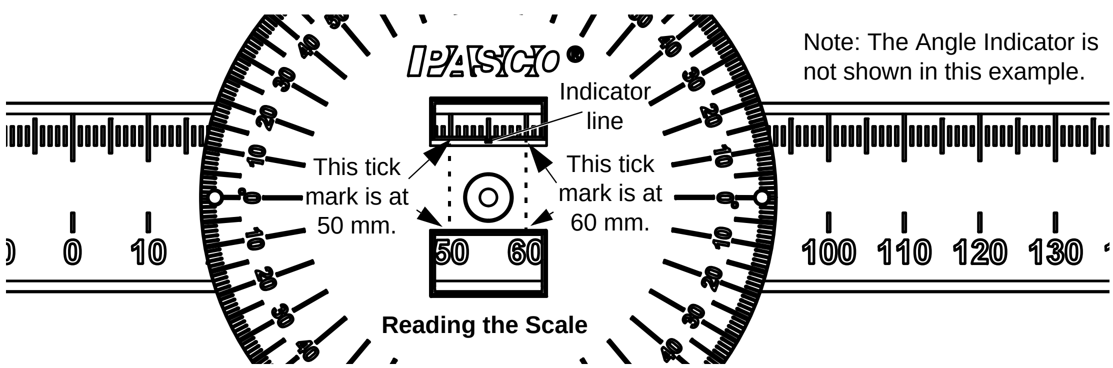


Balance Arm Assembly

Adjust the beam by loosening the thumbscrew on the top of the pivot and sliding the beam one way or the other. Move the protractors by loosening the thumbscrew at the center of the protractor and sliding the protractor along the beam. Note that not all experiments need three protractors.

Reading the Scale

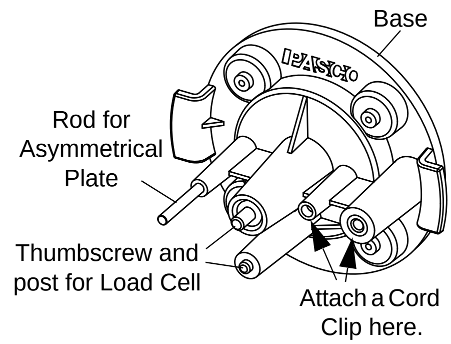
Read the position of the protractor relative to the midpoint through the two rectangular windows on the protractor. The bottom window shows the position to the nearest centimeter, and the top window shows the position to the millimeter. The top window has a small indicator line on its bottom edge. The bottom window in the example shows the position as between 50 mm and 60 mm and the top window shows it at 55 mm.



Reading the Scale

Utility Mount Assembly

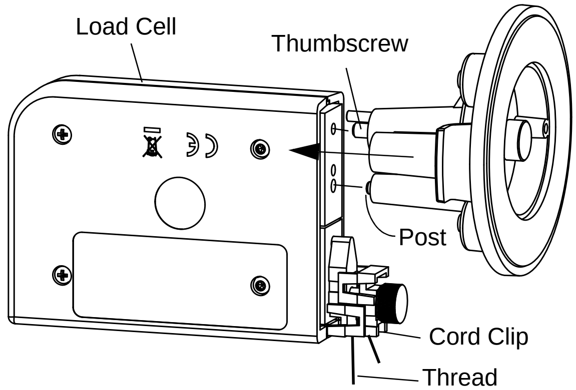
The Utility Mount has several functions. It can support a PS-2201 5 N Load Cell, it has a rod from which you can hang the Asymmetrical Plate, and the included Cord Tensioning Clip allows you to connect a thread to the mount. The thumbscrew for the Load Cell is “captured” by a rubber O-ring so that it will not fall out of the mount.



Utility Mount

Attaching a Load Cell

You can use a 5N Load Cell (available separately) with the Statics System to measure a force in much the same way that the Mounted Scale can be used to measure a force. (In fact, after the Load Cell is calibrated, it could be used to calibrate the Mounted Scale.) First, loop a thread through a Cord Clip (see below) and then attach the clip to the “LOAD” position that is imprinted on the side of the Load Cell. Next, attach the Load Cell’s “FIXED” end to the thumbscrew and post on the Utility Mount.



Attaching a Load Cell

Using a Cord Clip

The Cord Clip connects to the Utility Mount so that to a thread and be attached to the Mount itself. Use the Clip to attach a thread to a Load Cell (see above) that is mounted on the Utility Mount.

It is best to attach the thread through the clip before the clip is installed on the mount or on a Load Cell. The Cord Clip has two parts but it does not come apart. Start by holding the top part of the clip so the two parts are separate as shown in Figure A, leaving an opening through which the thread can be threaded. Insert the thread into the flat end (not the pointed end) of the clip. Loop the thread back through the clip as shown in Figure B and Figure C. Attach the Cord Clip to the Load Cell or the Utility Mount using the thumbscrew to tighten the clip shut.

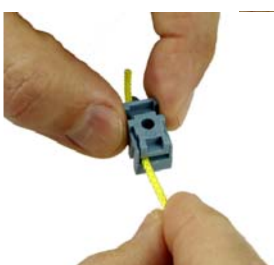


Figure A: Hold the cord clip so the two parts separate

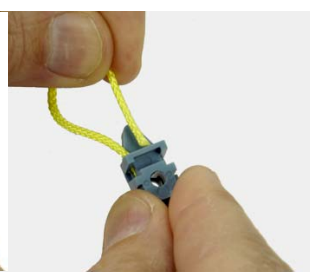


Figure B: Loop the thread back through the cord clip

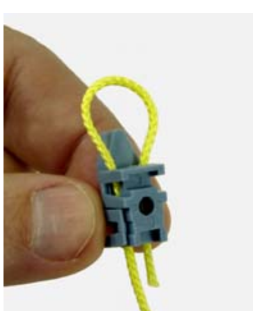


Figure C: The thread goes around the screw hole

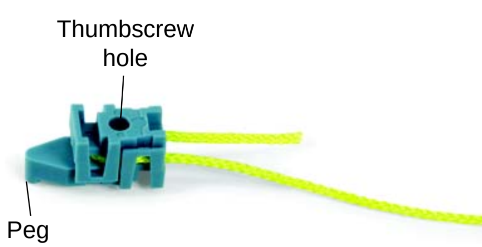
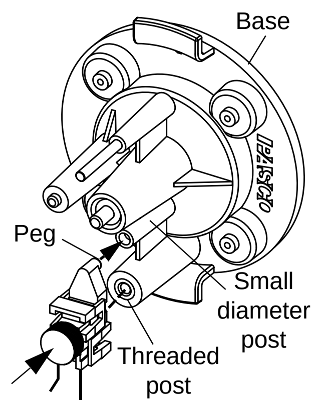


Figure D: The cord clip is ready to be attached to a Utility Mount or a Load Cell

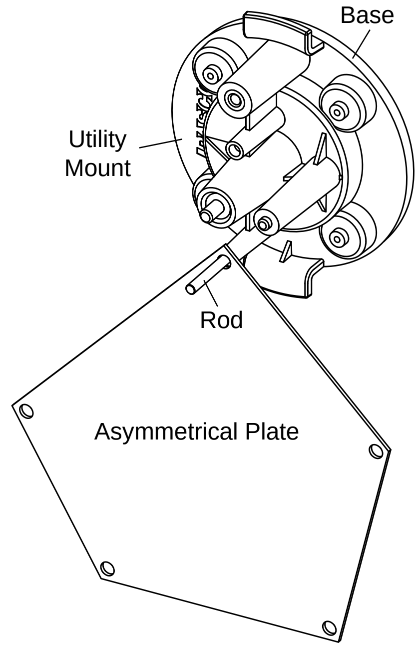
Attaching a Cord Clip



Align the peg of the Cord Clip with the hole in the small diameter post. Use the 5/8” thumbscrew to attach the Cord Clip to the threaded post.

Asymmetrical Plate

The Asymmetrical Plate is a five-sided polygon with a small hole at each corner. The Utility Mount has a slender metal rod designed to support the Asymmetrical Plate so that the plate can swing on the rod until it reaches equilibrium.



Suspend the Asymmetrical Plate

About the Manual

The ME-9502 Statics System provides an introduction to static mechanics. The experiments first introduce force as a vector quantity and then build on this concept so that students will understand the equilibrium of a physical body under the application of a variety of forces and torques. Experiments to demonstrate simple harmonic motion with a pendulum and a spring/mass system are also included.

The experiments are presented in three groups: Basic Experiments, Advanced Experiments, and Simple Machines. Each experiment is copy-ready.

Basic Experiments provide the essentials for a solid introduction to static mechanics. The concepts of vector forces, torques, and center of mass are explored.

Advanced Experiments allow the student to combine the principles already studied to understand such phenomena as static equilibrium. Friction and simple harmonic motion are also investigated.

Simple Machines provide an opportunity for the student to investigate applications of the principles already studied and also to introduce important new concepts. Levers, inclined planes, and pulley systems are studied using the principles of static equilibrium, work, and conservation of energy.

In addition to the equipment in the Statics System, a few common items such as pencils, rulers, protractors, paper, a stop watch, and a mass balance will be needed for some experiments. Check the “Equipment Needed” section at the beginning of each experiment.

NOTE: Vector quantities are designated with **boldface** font, such as **F**, **W**, or $\mathbf{F_1}$. When the same letters are used in normal style font, they refer to the magnitude (size) of the vector quantity. Since the vector nature of torques is not introduced in these experiments, boldface font is not used to designate torques.

Exp. 1: Hooke’s Law—Measuring Force

Equipment Needed

Item	Item
Statics Board	Mounted Spring Scale
Mass and Hanger Set	Thread

Introduction

At a practical level, a force is simply a push or a pull. A force is also a vector quantity that has magnitude (size) and direction. There are different ways to apply and measure a force. One way to apply a force is to hang a known mass, and determine the force based on the assumption that gravity pulls the mass downward toward the center of the earth with a magnitude $F = mg$, where m is the known mass and g is the acceleration due to gravity (9.8 m/s^2). Note that the value of g can also be expressed as 9.8 N/kg. Another way to apply a force is to pull with a spring. A spring stretches when it is pulled, and if he amount it stretches is directly proportional to the applied force, then the spring can be calibrated to measure unknown forces. In this experiment you will use the known force associated with gravity pulling on calibrated masses to investigate the properties of the Mounted Spring Scale.

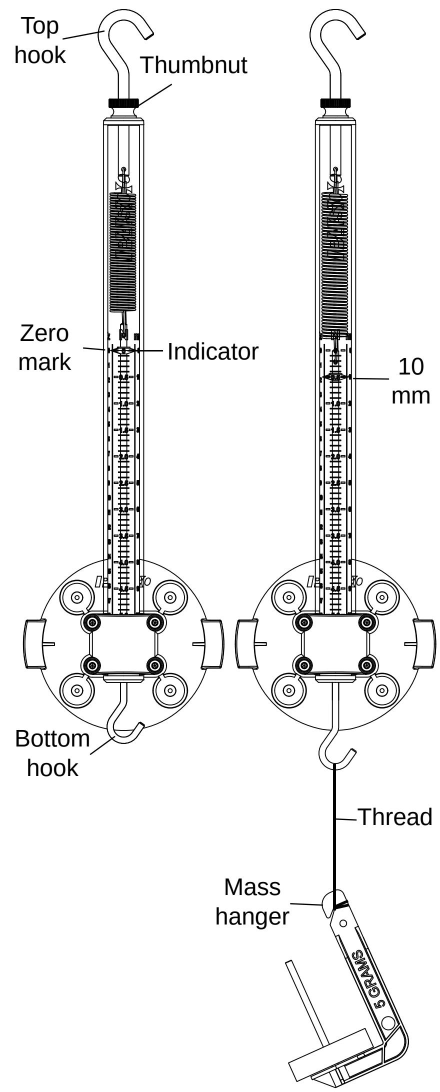


Figure 1.1: Setup and Procedure

Hooke’s Law

Hooke’s Law describes the relationship between the amount of force and the amount of stretch for an “ideal” spring. The law states that the force and the stretch are directly proportional. In other words, the ratio of the force divided by the stretch is a constant, k. The constant is called the “spring constant”.

Setup

Place the Spring Scale on the Statics Board so that the spring hangs vertically in the tube. Do not hang anything on the bottom hook. The indicator must be aligned with the zero mark on the label. To zero the Spring Scale, loosen the thumbnut at the top of the scale. Turn the top hook clockwise to lower the indicator, and counter-clockwise to raise the indicator. Once the indicator is aligned with the zero mark on the label, tighten the thumbnut.

Procedure

1. Attach a thread to the bottom hook and hang a mass hanger from the thread.
2. Add mass to the mass hanger until the indicator is aligned with the 10 mm mark on the label. Adjust the mass so that the indicator is as close as possible to the mark. Estimate the uncertainty in your measurement. (If you add or remove 0.5 g, can you see a change? What happens if you add or remove 1 g or 2 g?)
3. Record the total amount of mass (including the mass hanger) in the data table. Record the uncertainty.
4. Add more mass to the mass hanger until the indicator is aligned with the 20 mm mark on the label and record the total amount of mass and the uncertainty.
5. Repeat the process to move the indicator down by 10 mm each time until the indicator is aligned with the 80 mm mark. Record the total mass and the uncertainty each time.

Data Table

Spring Displacement (m)	Mass (kg)	Uncertainty	Weight (N)
0.010 m (10 mm)			
0.020 m (20 mm)			
0.030 m (30 mm)			
0.040 m (40 mm)			
0.050 m (50 mm)			
0.060 m (60 mm)			
0.070 m (70 mm)			
0.080 m (80 mm)			

Calculations

1. Using the formula $F = mg$, where m is the mass and g is the acceleration due to gravity, calculate the weight in newtons for each trial. Record the weight in the data table. (To get the correct force in newtons, you must convert the mass value to kilograms.)

2. On a sheet of graph paper, construct a graph of Weight (N) versus Spring Displacement (m) with Spring Displacement on the x-axis.

3. Draw the line that best fits your data points on the graph. The slope of this line on the graph is the ratio of the force that stretched the spring divided by the amount of stretch. In other words, the slope is the spring constant, k, for the spring in the Spring Scale.

4. Determine the spring constant, k, from your graph and record the result. Remember to include the units (newtons per meter).

Spring constant = ______________________________

Using a Spring Scale to Measure Force

- Hang 160 g (0.160 kg) on the Spring Scale. Calculate the weight based on $\boldsymbol{F = mg}$. Read the force in newtons from the Spring Scale.

Weight = ________________ Spring Scale reading = _______________

- How does the measurement from the Spring Scale compare to the actual weight?

- Calculate the percent difference: $\left|\frac{\text{Weight - Spring Scale}}{\text{Weight}}\right| X100$

Percent Difference = ________________

Questions

1. Hooke's Law states that the relationship between force and displacement in springs is a linear relationship. If Hooke's Law was not valid, could a spring still be used successfully to measure forces? If so, how?

2. In what way is Hooke's Law useful when calibrating a spring for measuring forces?

3. On your graph of Weight versus Spring Displacement, did the best fit line go through the origin (zero)? If it didn't, what does that mean?

Exp. 2: Adding Forces—Resultants and Equilibriants

Equipment Needed

Item	Item
Statics Board	Mounted Spring Scale
Force Wheel	Large and Small Pulleys
Mass and Hanger Set	Thread

Theory

In figure 2.1, Person A and Person B are pulling with forces represented by $\mathbf{F_A}$ and $\mathbf{F_B}$ on a car stuck in the mud. Since these forces are acting on the same point of the car, they are called **concurrent** forces. Each force is defined both by is direction (the direction of the vector arrow), and by its magnitude, which is proportional to the length of the vector arrow. (The magnitude of the force is independent of the length of the tow rope.)

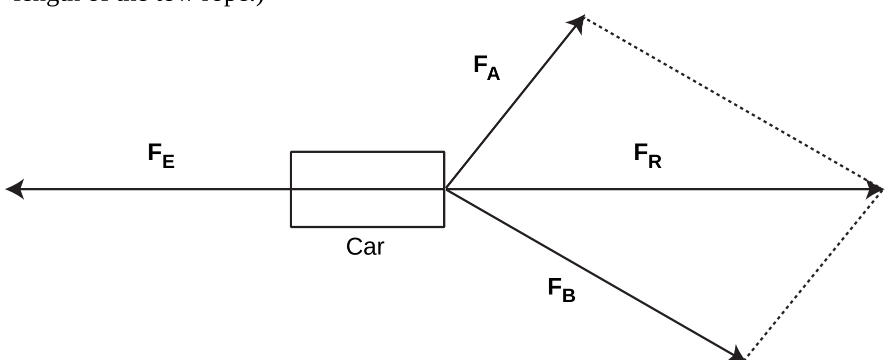


Figure 2.1: Resultant and Equilibriant

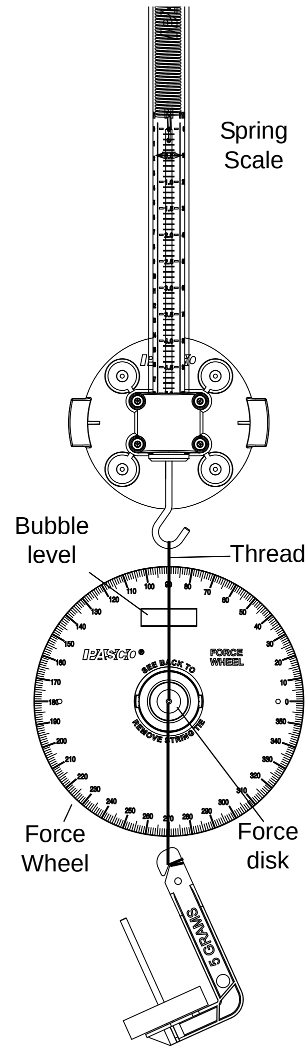
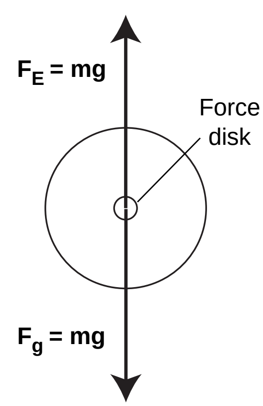


Figure 2.2: Setup



The total force applied by the two people can be determined by adding vectors $\mathbf{F_A}$ and $\mathbf{F_B}$. The parallelogram method is used in the example. The diagonal of the parallelogram is called the **resultant**, $\mathbf{F_R}$. It shows the magnitude and direction of the combination of $\mathbf{F_A}$ and $\mathbf{F_B}$.

Because the car is not moving, the net force on the car must be zero. The friction between the car and the mud is equivalent to the resultant force, $\mathbf{F_R}$. This equivalent opposing force is called the **equilibriant**, $\mathbf{F_E}$. This force has the same magnitude as the resultant, $\mathbf{F_R}$, and it has the opposite direction of the resultant.

Setup

Set up the Spring Scale and Force Wheel on the Statics Board as shown. Twist the Force Wheel until the bubble level shows that the Force Wheel is level. Attach one of the threads from the force disk (inner part of the String Tie) in the center of the Force Wheel to the bottom hook of the Spring Scale. Connect a second thread to a mass hanger (let the third thread dangle). Add 80 g (0.080 kg) to the mass hanger.

Adjust the Spring Scale up or down so that the force disk is centered in the Force Wheel. The mass hanger applies a force downward, $\mathbf{F_g}$ = m**g** (the force due to gravity, where m is the total mass of the mass hanger). When the force disk is centered, the system is in equilibrium, so the downward force $\mathbf{F_g}$ must be balanced by an equal and opposite force, the **equilibriant**, $\mathbf{F_E}$. In this case, the equilibriant force, $\mathbf{F_E}$, is applied by the Spring Scale.

Procedure: Two Forces

1. Add or remove 0.5 g to the mass hanger. Did the force disk move away from the center position? How much can you change the mass on the mass hanger without changing where the force disk is centered?

2. What are the magnitude and direction of $\mathbf{F_g}$, the gravitational force applied by the hanger, where $\mathbf{F_g} = m\mathbf{g}$?

- $\mathbf{F_g}$: Magnitude ________________ Direction ________________

3. Use the Spring Scale and Force Wheel to determine the magnitude and direction of $\mathbf{F_E}$, the equilibriant.

- $\mathbf{F_E}$: Magnitude ________________ Direction ________________

Procedure: Three Forces

1. Attach the Large Pulley and the two Small Pulleys to the Statics Board and move the Spring Scale as shown in the figure.

2. Attach threads to the bottom hook of the Spring Scale and to mass hangers over the Small Pulleys.

3. Add 30 g (0.030 kg) to the upper mass hanger and add 50 g (0.050 kg) to the lower mass hanger.

4. Adjust the Large Pulley and the Spring Scale so that the force disk is centered in the Force Wheel.

5. How much can you change the mass on the hangers and still leave the force disk centered in the Force Wheel?

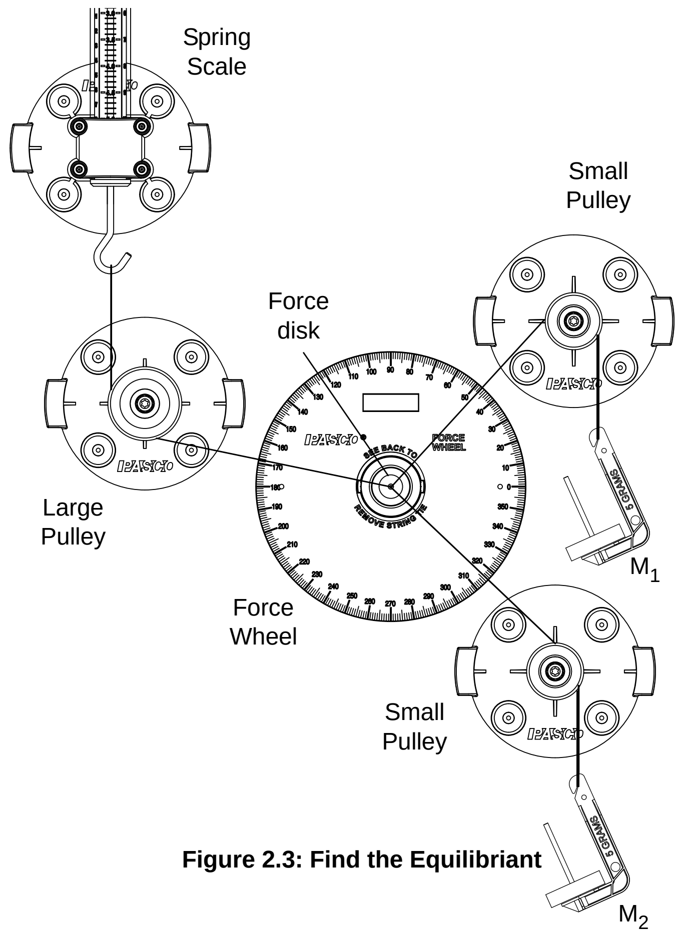
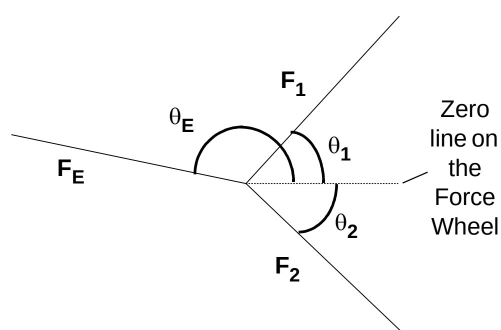


Figure 2.3: Find the Equilibriant



Vector diagram

Data

Record the values of the hanging masses M_1 and M_2 (including the mass of the mass hangers), the magnitude in newtons of the forces $\mathbf{F_1}$, $\mathbf{F_2}$, and $\mathbf{F_E}$, and the angles θ_1, θ_2, and θ_E with respect to the zero-degree line on the Force Wheel..

Mass (kg)	Force (N)	Angle (°)
M_1	F_1	θ_1
M_2	F_2	θ_2
	F_E	θ_E

Analysis

1. On a separate piece of graph paper, use the values you recorded in the table to construct a vector diagram for $\mathbf{F_1}$, $\mathbf{F_2}$, and $\mathbf{F_E}$. Choose an appropriate scale, such as 2.0 centimeters per newton, and make the length of each vector proportional to the magnitude of the force. Label each vector and indicate the magnitude of the force it represents.

2. On your vector diagram, use the parallelogram method to draw the resultant of $\mathbf{F_1}$ and $\mathbf{F_2}$. Label the resultant $\mathbf{F_R}$. Measure the length of $\mathbf{F_R}$ to determine the magnitude of the resultant force and record this magnitude on your vector diagram.

3. On your diagram, measure the direction of $\mathbf{F_R}$ relative to the horizontal axis of your diagram and label the angle $\theta_\mathbf{R}$.

Questions

1. Does the magnitude of the equilibriant force vector, $\mathbf{F_E}$, exactly balance the magnitude of the resultant force vector, $\mathbf{F_R}$. If not, what are some possible reasons for the difference?

2. How does the direction of the equilibriant force vector, $\mathbf{F_E}$, compare to the direction of the resultant force vector, $\mathbf{F_R}$?

Extension

Vary the magnitudes and directions of $\mathbf{F_1}$ and $\mathbf{F_2}$ and repeat the experiments.

Exp. 3: Resolving Forces—Components

Equipment Needed

Item	Item
Statics Board	Mounted Spring Scale
Force Wheel	Pulleys (2)
Mass and Hanger Set	Thread

Theory

In experiment 2, you added concurrent forces vectorially to determine the magnitude and direction of the combined forces. In this experiment, you will do the reverse: you will find two forces which, when added together, have the same magnitude and direction as the original force. As you will see, any force vector in the X-Y plane can be expressed as a vector in the x-direction and a vector in the y-direction.

Setup

Set up the equipment as shown in the figure. As shown, create a force vector **F** by hanging a mass, M_1, on a thread from the force disk in the center of the Force Wheel over a pulley.

Set up the Spring Scale and a pulley so that the thread from the Spring Scale is horizontal from the bottom of the pulley to the force disk. Hang a second mass hanger directly from the force disk in the center of the Force Wheel.

Now pull the Spring Scale toward or away from the pulley to adjust the horizontal or x-component of the force, $\mathbf{F_x}$. Adjust the amount of mass on the vertical mass hanger, M_2, to adjust the vertical or y-component of the force, $\mathbf{F_y}$. Adjust the x- and y-components in this way until the force disk is centered in the Force Wheel.

Note: These x- and y-components are actually the x- and y-components of the equilibriant, $\mathbf{F_E}$, of the force **F**, rather than the components of **F** itself.

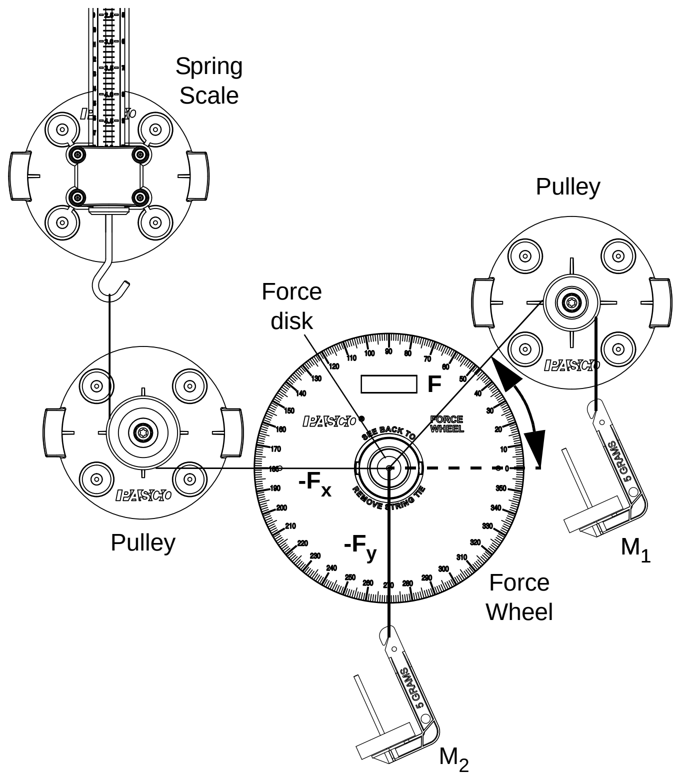


Figure 3.1: Setup

Procedure 1

1. Calculate and record the magnitude of **F** (based on **F** = m**g**). Use the Force Wheel to measure the angle of **F** and then record the angle.

- Magnitude, F = __________________ Angle, θ = _____________________

2. Record the magnitude of the x-component of the equilibriant of **F** and calculate and record the y-component of the equilibriant of **F**.

- X-component = __________________ Y-component = _____________________

3. What are the magnitudes of $\mathbf{F_x}$ and $\mathbf{F_y}$, the x- and y-components of **F**?

- $\mathbf{F_x}$ = ________________ $\mathbf{F_y}$ = ____________________

4. Change the magnitude and direction of the force vector F and repeat the experiment.

5. Record the magnitude and angle of the new force vector, **F**, and the magnitudes of $\mathbf{F_x}$ and $\mathbf{F_y}$.

- Magnitude, F = ________________ Angle, θ = ____________________
- $\mathbf{F_x}$ = ________________ $\mathbf{F_y}$ = ____________________

Background

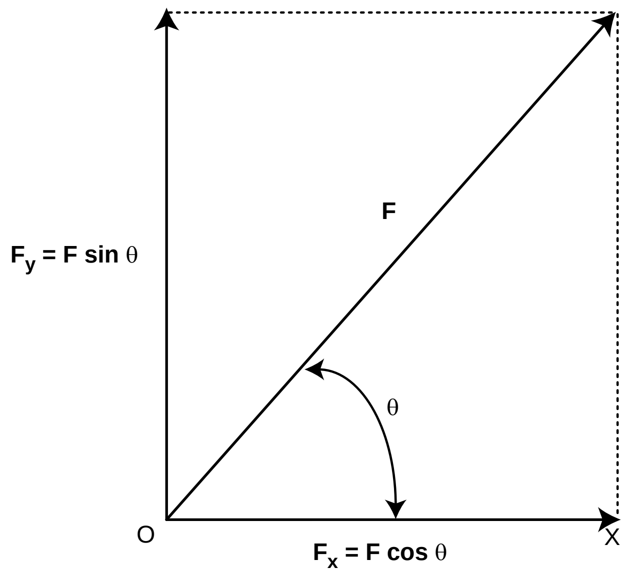


Figure 3.2: Vector components

Why use components to specify vectors? One reason is that using components makes it easier to add vectors mathematically. The figure shows the x- and y-components of a vector of length F, and an angle θ with respect to the x-axis. Since the components are at right angles to each other, the parallelogram used to determine their resultant is a rectangle. Using right triangle AOX, the components of F can be calculated:

- $\mathbf{F_x}$, the x-component of **F** is F cos θ
- $\mathbf{F_y}$, the y-component of **F** is F sin θ

If you want to add several vectors, find the x- and y-components for each vector. Add all the x-components together and all the y-components together. The resulting values are the x- and y-components for the **resultant**. The magnitude of the resultant, F_R, is the square root of the sum of the squares of the resultant's x-component (R_x) and the y-component (R_y). $F_R = \sqrt{(R_x)^2 + (R_y)^2}$

The angle of the resultant is the arctangent of the y-component divided by the x-component. $\theta_R = \operatorname{atan}\left(\frac{R_y}{R_x}\right)$

Procedure 2

Make sure that the Force Wheel is level. Set up a new force, **F**. Put one thread from the force disk over a pulley and tie a mass hanger to the end of the thread. Add mass to the mass hanger.

1. Calculate and record the magnitude of the force, **F**, that you set up with the pulley and mass hanger. Use the Force Wheel to determine the angle.

- Magnitude, F = ________________ Angle, θ = ____________________

2. Calculate the magnitudes of $\mathbf{F_x}$ and $\mathbf{F_y}$, the x- and y-components of the new force, **F**. (Remember, F_x = F cos θ and F_y = F sin θ.)

- $\mathbf{F_x}$ = ________________ $\mathbf{F_y}$ = ____________________

3. Now, set up the Spring Scale with a pulley and a thread from the force disk so it can apply the x-component, $\mathbf{F_x}$, of the new force **F**. Adjust the Spring Scale so that it pulls the force disk horizontally by the amount, $\mathbf{F_x}$.

4. Next, attach a mass hanger to the third thread from the force disk so the thread hangs vertically. Add mass to the hanger so it pulls the force disk vertically by the amount, $\mathbf{F_y}$.

Question

1. Is the force disk at equilibrium in the center of the Force Wheel?

2. Why or why not?

Extensions

1. Generally it is most useful to find the components of a vector along perpendicular axes, as you did above. However, it is not necessary that the axes be perpendicular. If time permits, try setting up the equipment to find the components of a vector along non-perpendicular axes. Use pulley to redirect the component forces to non-perpendicular directions.

2. The figure shows a classic vector combination. For the force disk to be in equilibrium, the x-components of force $\mathbf{F_1}$ and force $\mathbf{F_2}$ must be equal and opposite, and the y-components of $\mathbf{F_1}$ and $\mathbf{F_2}$ must add to equal the magnitude of force $\mathbf{F_3}$, the vertical force. Set up the equipment so that the Spring Scale applies force $\mathbf{F_1}$. Once the system is in equilibrium, determine the x- and y-components of the vectors and compare them.

3. For the setup in the figure, change the angles of $\mathbf{F_1}$ and $\mathbf{F_2}$2 so that the vectors are closer to the x-axis. Calculate the x-components and notice the change. What happens to the x-components as the two forces become closer and closer to parallel? What amount of force would be needed if $\mathbf{F_1}$ and $\mathbf{F_2}$ were both horizontal?

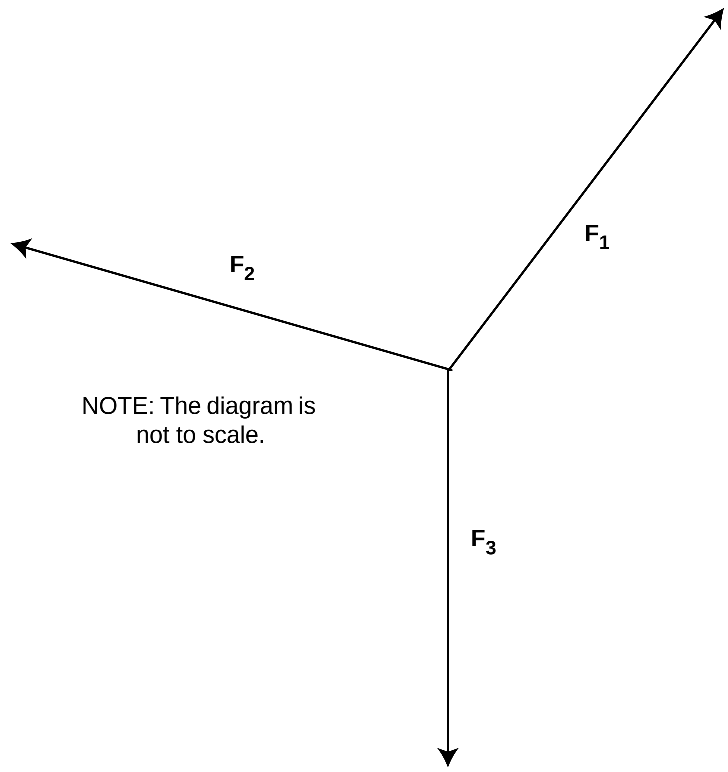


Figure 3.3: Extension vectors

Exp. 4: Torque—Parallel Forces

Equipment Needed

Item	Item
Statics Board	Balance Arm and Protractors
Mass and Hanger Set	Thread

Theory

In experiment 2, you found resultants and equilibriants for concurrent forces—forces that act upon the same point. In the real world, forces are often not concurrent. They act upon different points on an object. In the figure, for example, two forces are pulling on different points of an object. Two questions can be asked:

1. In which direction will the object be accelerated?

2. Will the object rotate?

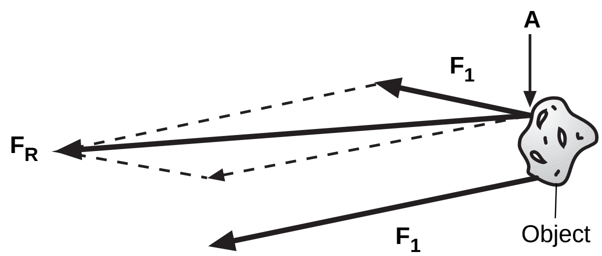


Figure 4.1: Non-concurrent forces

If the two forces were both applied at point **A**, the resultant would be the force vector shown, $\mathbf{F_R}$. In fact, $\mathbf{F_R}$ points in the direction in which the object will be accelerated. (This idea will be investigated further in later experiments,) What about question 2? Will the object rotate? In this experiment you will begin to investigate the types of forces that cause rotation in physical bodies. In doing so, you will encounter a new concept—torque.

Setup

Mount the Balance Arm near the center of the Statics Board.

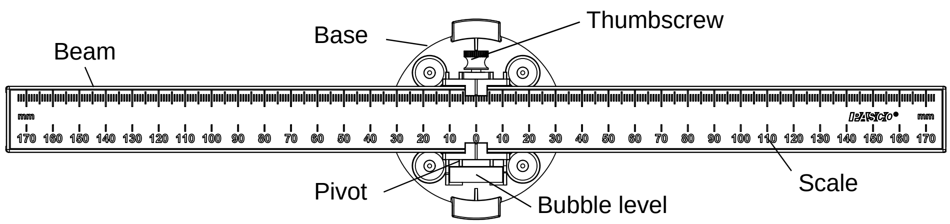


Figure 4.2: Balance Arm

Level the Beam

Loosen the thumbscrew and adjust the beam so that the zero mark on the beam is aligned with the indicator marks on the pivot. When the beam is level, the bubble in the bubble level will be midway between the two lines on the level.

Add the Protractors

First, find the mass of two of the protractors and record the masses. (Note that you can use the Spring Scale to measure the mass.)

- Protractor 1 = __________
- Protractor 2 = __________

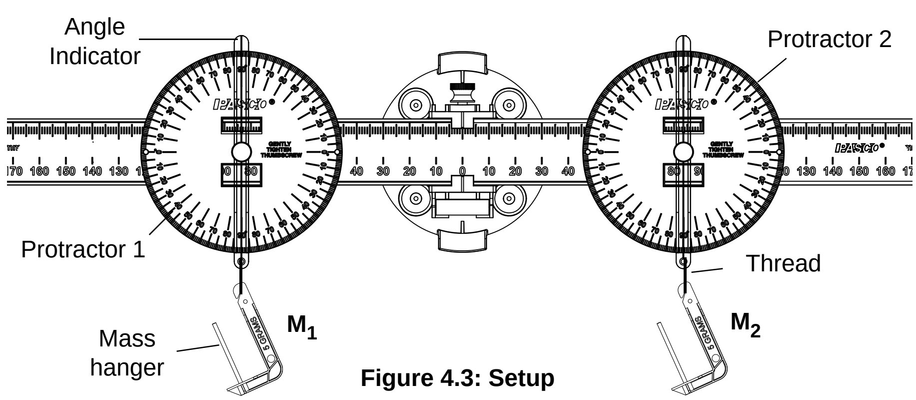


Figure 4.3: Setup

Loosen the thumbscrews on the two protractors and slide one onto each end of the beam. Tie a mass hanger to the thread on the angle indicator of each protractor.

Procedure: Equal Distance, Equal Mass

Position one of the protractors near one end the beam and tighten its thumbscrew to hold it in place. Adjust the position of the other protractor until the beam is perfectly balanced, and then tighten its thumbscrew to hold it in place.

1. Measure d_1 and d_2, the distances from the pivot to the center of each protractor.

- d_1 = ________________
- d_2 = ________________

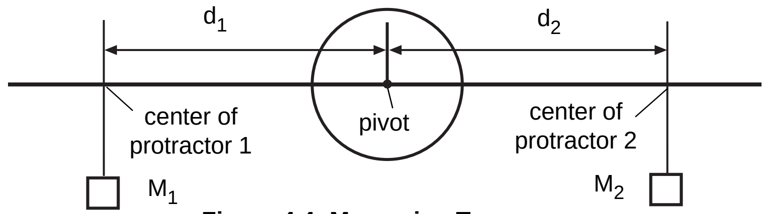


Figure 4.4: Measuring Torques

2. Add a 50-gram mass to each mass hanger.

- Is the beam still balanced?

3. Add an additional 20-gram to one of the mass hangers.

- Can you restore the balance of the beam by repositioning the other protractor and mass hanger?

Procedure: Unequal Distance, Unequal Mass

Position one of the protractors approximately halfway between the pivot point and the end of the beam and tighten its thumbscrew to hold it in place. Add 75 grams of mass to the mass hanger, M_1.

Place various masses on the other mass hanger (M_2) and slide it along the beam as needed to rebalance the beam.

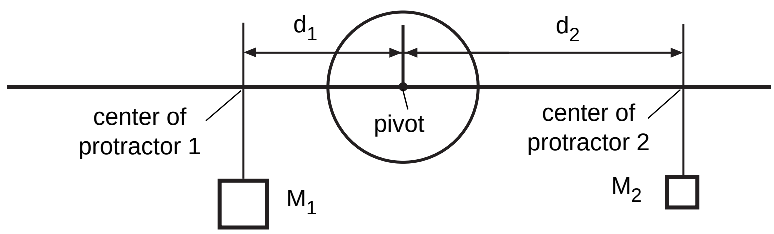


Figure 4.5: More Torques

1. At the first balanced position, measure the total mass, M_1 and M_2, on each side of the pivot (protractor, mass hanger, added masses) and record the masses in the data table.

2. Measure the distances, d_1 and d_2, between the centers of the protractors and the pivot and record the values in the data table.

3. Take measurements for five more different values of M_2 and record your results in the data table. Be sure to include the units of your measurements.

4. If there is time, vary M_1 and repeat the procedure.

- Reminder: For accurate results, include the mass of the protractor, mass hanger, and added masses when measuring M_1 and M_2.

Calculations

Calculate the gravitational force (weight = m**g**) produced by the total mass on each side of the beam for each case. Calculate the torques, τ_1 and τ_2, on each side of the beam for each case. Remember, torque, τ, is the cross product of the net force and the lever arm. Since the distance and the direction of the force are at right angles in this experiment, the torque, τ, is F d (where $\mathbf{F_g} = m\mathbf{g}$). Record your calculated values of weight, $\mathbf{F_g}$, and torque, τ, for each balanced position of the beam.

Data Table

Case	Total Mass M_1 (kg)	Weight F_1 (N)	Distance d_1 (m)	Torque $\tau_1 = F_1 d_1$	Total Mass M_2 (kg)	Weight F_2 (N)	Distance d_2 (m)	Torque $\tau_2 = F_2 d_2$
1								
2								
3								
4								
5								
6								

Questions

1. From your results, what mathematical relationship must there be between τ_1 and τ_2 in order for the beam to be balanced?

2. What torque is exerted on the balance beam by the upward pull of the pivot point?

Extension

If you have time, try adding a third Protractor and Mass Hanger to the beam on the same side of the beam as the second Protractor.

Question

What relationship must there be between τ_1, τ_2, and τ_3 in order for the beam to be balanced when there are three protractors and mass hangers?

Exp. 5A: Center of Mass

Equipment Needed

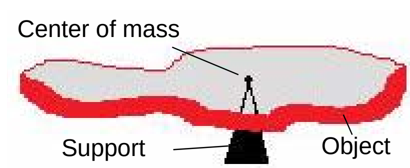
Item	Item
Statics Board	Balance Arm and Protractors
Asymmetrical Plate	Thread
Mass and Hanger Set	

Theory

Gravity is a universal force; every bit of matter in the universe is attracted to every other bit of matter. Therefore, when the Balance Arm is supported by the pivot, every bit of matter in the Balance Arm is attracted to every bit of matter in the Earth.

Fortunately, the sum of all these individual gravitational forces produces a single resultant. This resultant acts as if it were pulling between the center of the Earth and the center of mass of the Balance Arm. The magnitude of the force is the same as if all the matter of the Earth were located at the center of the Earth, and all the matter of the Balance Arm were located at the center of mass of the Balance Arm.

An object thrown so that it rotates tends to rotate about its center of mass, and the center of mass follows a parabolic path. An object whose center of mass is above a support tends to remain in rotational equilibrium (balanced on the support). In this experiment you will use your knowledge of torque to understand and locate the center of mass of an object.



Setup

Measure and record the mass of the beam.

Next, find the mass of two of the protractors and record the masses. (Note that you can use the Spring Scale to measure the mass.)

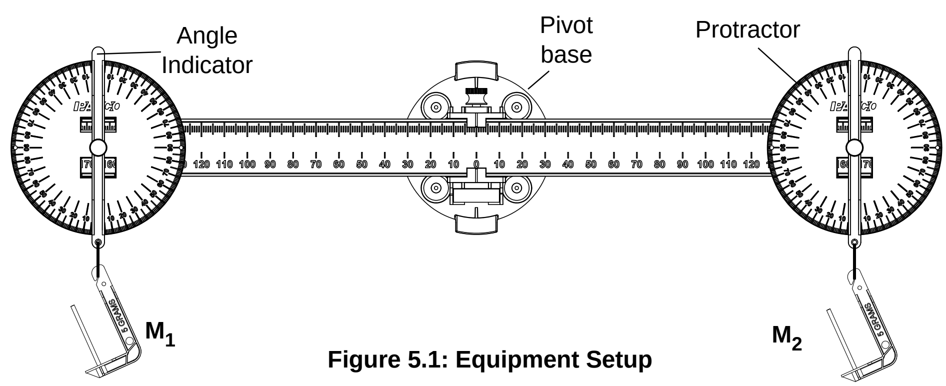


Figure 5.1: Equipment Setup

- Beam = ________
- Protractor 1 = __________
- Protractor 2 = __________

Loosen the thumbscrews on the two protractors and slide one onto each end of the beam so each protractor is at the 165 mm mark. Tie a mass hanger to the thread on the angle indicator of each protractor

Mount the Balance Arm near the center of the Statics Board.

Record the total mass of the beam plus protractors plus mass hangers (5 grams each).

- Mass of system = _____________

Level the Beam and Mark the Center of Mass

Loosen the thumbscrew and adjust the beam so that the indicator marks on the pivot are aligned with the zero mark on the beam. If necessary, adjust the positions of the two protractors until the bubble in the bubble level is midway between the two lines on the level. Once the beam is balanced and level, tighten the thumbscrews to hold the protractors and beam in place.

Put a pencil mark on the beam to indicate the center of mass of the system. All of the mass of the system (beam, protractors, mass hangers) acts as if it is concentrated at the center of mass.

Once the beam is in balance, the force at the pivot point must be the equilibriant of the total gravitational force acting on the beam. Since the beam does not rotate, the gravitational force and its equilibriant must be concurrent forces.

Experiment

1. Why would the Balance Arm necessarily rotate if the resultant of the gravitational forces and the force provided by the pivot were not concurrent forces?

- Think of the Balance Arm as a collection of many small hanging masses. Each hanging mass is pulled down by gravity and therefore produces a torque about the pivot point of the Balance Arm.

2. What is the relationship between the sum of the clockwise torques about the center of the mass and the sum of the counterclockwise torques about the center of mass? Explain.

- Add 50 grams to one mass hanger and 100 grams to the other mass hanger. Loosen the thumbscrew on the beam and slide the beam through the pivot until the beam and masses are balanced. Tighten the thumbscrew.

- The pivot is still supporting everything (beam, protractors, mass hangers, and hanging masses), but at the new center of mass of the system–the pivot point.

3. Calculate the three torques, τ_1, τ_2, and τ_3 provided by the three forces F_1, F_2, and F_3 acting about the new pivot point position. Be sure to indicate whether each torque is clockwise (cw) or counterclockwise (ccw).

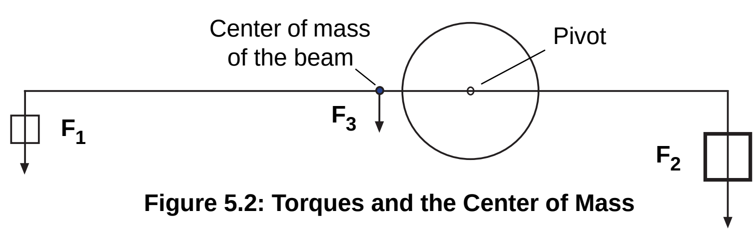


Figure 5.2: Torques and the Center of Mass

Position	Mass (kg)	Force (F = mg)	Distance (m)	Torque (τ = F d)
1	0.050			
2	0.100			
3				

4. Are the clockwise and counterclockwise torques balanced?

- Remove the 50 gram mass from the left hand mass hanger, but leave the hanger and protractor in place. (Removing the mass effectively removes F_1). Reposition the beam in the pivot until the torque provided by F_3 balances the torque provided by F_2 and the beam is level again.

- Recalculate the torques about the pivot point.

Position	Force (F = mg)	Torque (τ = F d)
2		
3		

5. Are the torques balanced?

Asymmetrical Plate

Replace the Balance Arm with the Force Sensor Mount. Hang the Asymmetrical Plate from the rod on the Force Sensor Mount.

Since the force of the rod acting on the plate is the equilibriant to the sum of the gravitational forces acting on the plate, the line of force exerted by the rod must pass through the center of the mass of the plate. Loop a piece of thread over the rod and attach a mass hanger to the end of the thread.

Use a pencil or a “dry erase” pen to draw a line on the Asymmetrical Plate that marks the position of the thread on the plate.

Remove the thread and mass hanger. Hang the plate from a different hole. Put the thread and mass hanger back on the rod. Draw a new line on the plate that marks the position of the thread.

Repeat the process for a third different hole. Draw a third line on the plate.

6. Does the line showing where the thread is hanging pass through the center of mass of the plate?

7. Would this method work for a three dimensional object? Why or why not?

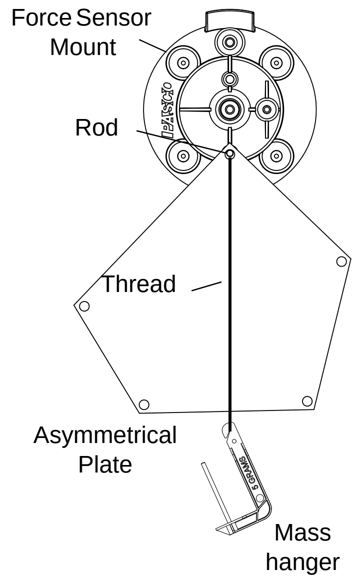


Figure 5.3: Finding the Center of Mass

Extension

Remove the thread and plate from the rod. Try to balance the Asymmetrical Plate on your finger by placing the tip of you finger under the point where the three drawn lines intersect. What happens?

Exp. 5B: Equilibrium of Physical Bodies

Equipment Needed

Item	Item
Statics Board	Balance Arm and Protractors
Pulley (1)	Mounted Spring Scale
Mass and Hanger Set	Thread

Theory

Any force acting on a body may produce both translational motion (movement of the center of mass of the body in the direction of the force) and rotational motion (rotation about a pivot point).

In this part of the experiment you will investigate the interplay between forces and torques by examining all the forces acting on a body in physical equilibrium.

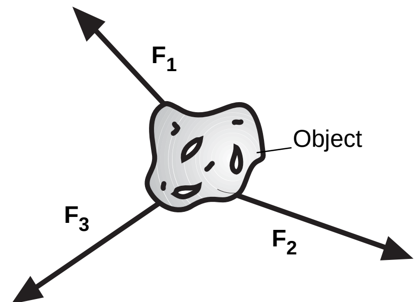


Figure 5.4: Non-concurrent, non-parallel forces

Setup

Find the center of mass of the balance beam and mark it with a pencil. Use the Spring Scale, mass hangers, and three protractors on the Balance Arm to set up the equipment as shown.

By supporting the Balance Arm from the Spring Scale, you can now determine all the forces acting on the Balance Arm.

As shown in the diagram, these forces include $\mathbf{F_1}$, the weight of the mass, $\mathbf{M_1}$, $\mathbf{F_2}$, the weight of the mass $\mathbf{M_2}$, $\mathbf{F_3}$, the weight of the balance beam acting through its center of mass, and $\mathbf{F_4}$, the upward pull of the Spring Scale.

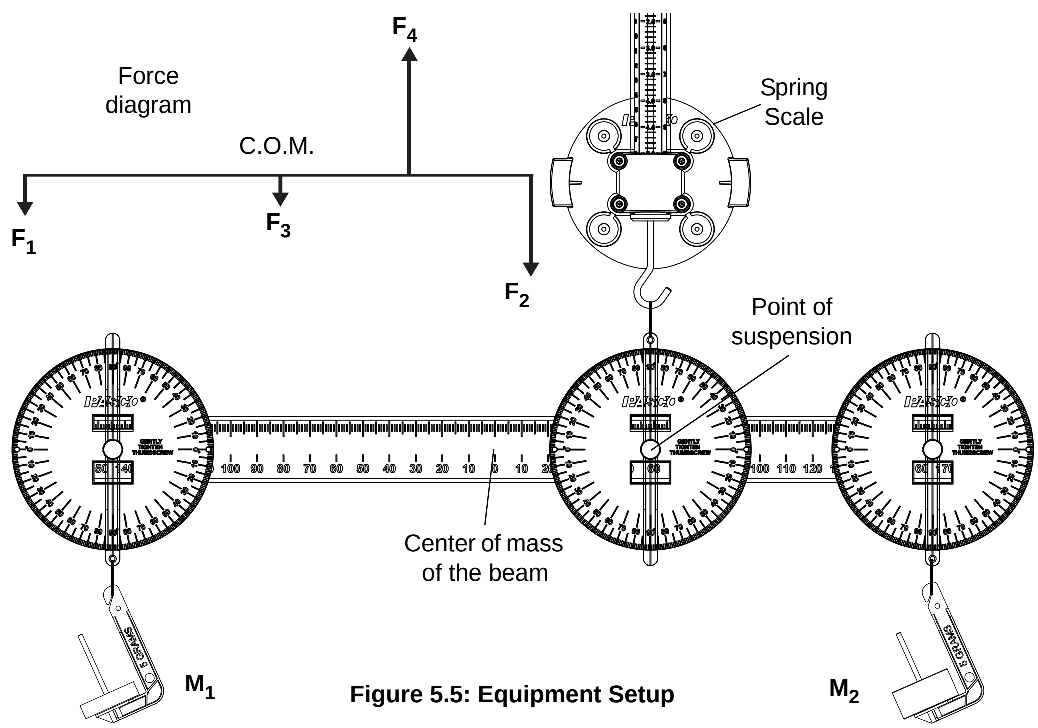


Figure 5.5: Equipment Setup

Experiment

Fill in the data table listing M (in kilograms), F (in newtons), d (the distance in meters from the applied force to the suspension point), and τ (the torque acting about the point of suspension in newton • meters. Indicate whether each torque is clockwise (cw) or counterclockwise (ccw)

Data Table.

Position	mass (M)	force (F)	distance (d)	torque (τ)
1				
2				
3				
4	—			

Calculations

1. Calculate and record the sum of the clockwise and counterclockwise torques.

$\Sigma\tau_{\mathbf{cw}}$ = ________________________ $\Sigma\tau_{\mathbf{ccw}}$ = ________________________

- Are the torques balanced?

2. Calculate and record the sum of the upward and downward forces.

$\Sigma F_{\mathbf{up}}$ = ________________________ $\Sigma F_{\mathbf{down}}$ = ________________________

- Are the translational forces balanced?

3. On the basis of your answers to the questions, what conditions must be met for a physical body to be in equilibrium (no acceleration)?

Change the Origin

In measuring the torques the first time, all the distances were measured from the point of suspension of the Balance Arm. This measures the tendency of the beam to rotate about this point of suspension. You can also measure the torques about any other point, on or off the beam. Using the same forces as you used before, re-measure the distances, measuring from the left end of the balance beam as shown in the diagram.

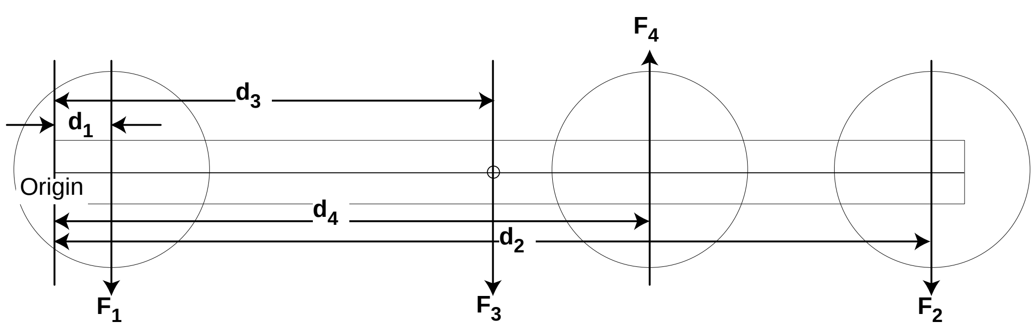


Figure 5.6: Change the Origin

Recalculate the torques to determine the tendency of the beam to rotate about the left end of the beam.

Record your data in the second table. As before, indicate whether each torque is clockwise (cw) or counterclockwise (ccw).

Data Table: Change the Origin.

Position	force (F)	distance (d)	torque (τ)
1			
2			
3			
4			

Calculations

1. Calculate and record the sum of the clockwise and counterclockwise torques.

$\Sigma\tau_{\mathbf{cw}}$ = ___________________________ $\Sigma\tau_{\mathbf{ccw}}$ = _________________________

- Are the torques balanced?

Extension

Use a pulley, hanging mass, and a thread to produce an additional upward force at one end of the beam. You may need to use tape to secure the thread to the beam.) Adjust the positions of the remaining hanging masses and the Spring Scale to bring the beam back into balance so it is level horizontally.

- Are all the forces balanced, both for translational and rotations motion?
- Diagram your setup and show your calculations on a separate sheet of paper.

Exp. 6: Torque—Non-Parallel Forces

Equipment Needed

Item	Item
Statics Board	Balance Arm and Protractors
Pulley	Mounted Spring Scale
Mass and Hanger Set	Thread

Theory

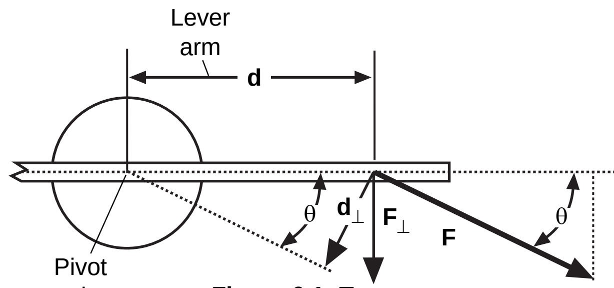


Figure 6.1: Torque

In a previous experiment, you investigated torques appled to the Balance Arm and discovered that when the torques about the point of rotation are balanced, the beam remains balanced. All the forces in that experiment were perpendicular to the beam and parallel to each other. What happens when one or more of the forces is *not* perpendicular to the beam?

It turns out that the formula for torque can be generalized to account for this case. Torque is the cross product of the force vector and the lever arm where the lever arm is the distance from the pivot point to where the force is applied. The generalized formula is:

$$\tau = Fd\sin\theta$$

where *F* is the magnitude of the force vector, *d* is the distance from the pivot point to the point at which the force is applied (that is, the “lever arm”), and θ is the angle between the force vector, ***F***, and the lever arm, *d*. Note that *F* sin θ is $F_\perp$, the component of the force vector, ***F***, that is perpendicular to the lever arm, *d*. Note also that *d* sin θ is $d_\perp$, the component of the lever arm, *d*, that is perpendicular to the force vector, ***F***. In other words, *d* sin θ is the perpendicular distance, $d_\perp$, from the pivot point to the line of force.

Setup

Mount the Balance Arm on the left half of the Statics Board. Adjust the beam so that the zero marks on the beam align with the indicator marks on the pivot.

Put a protractor on each end of the beam (for example, between 130 and 140 mm from the pivot). Adjust the position of the protractors if necessary so that the beam is balanced and level.

Suspend a mass, M_2, from one protractor. Mount the Spring Scale on the Statics Board and connect it to the other protractor using a thread and a Pulley as shown.

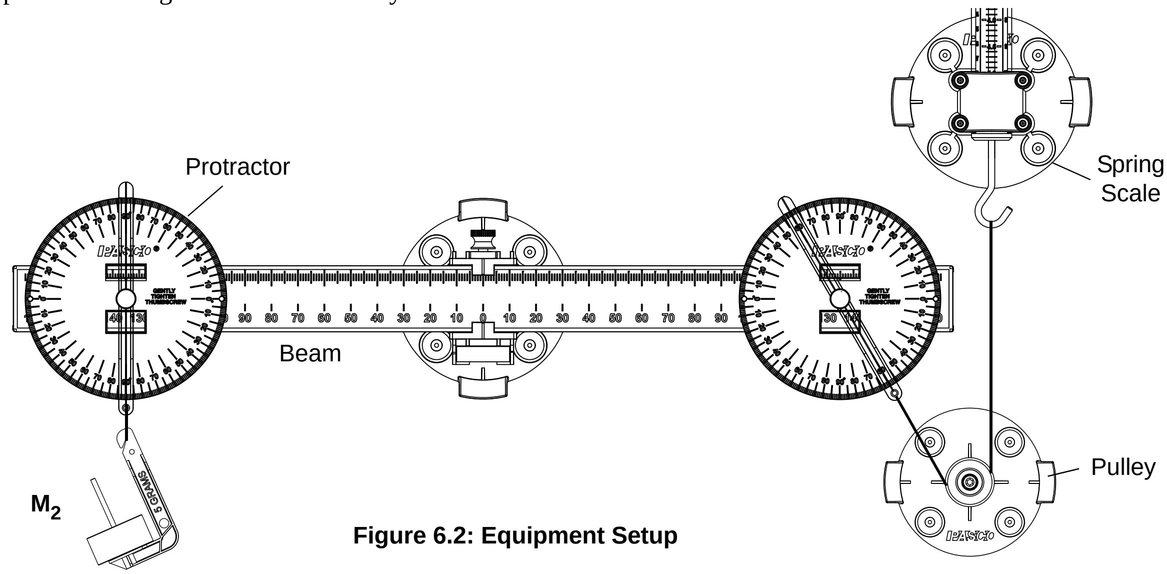


Figure 6.2: Equipment Setup

Procedure

1. Measure d_1 and d_2 and record the values.

2. Record the total mass, M_2, and the magnitude of the force, $\mathbf{F_2}$ (weight of the hanging mass). Use your measured values to calculate the torque, τ_2, produced by the force, $\mathbf{F_2}$. Be sure to include the units for each value.

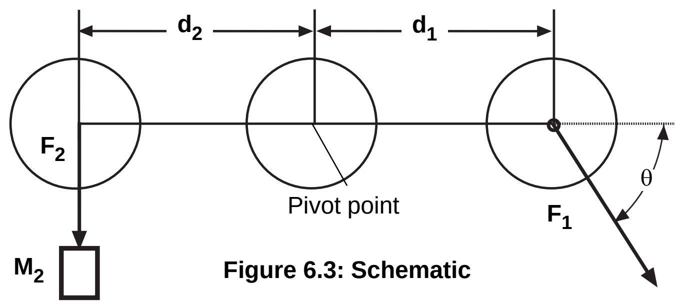


Figure 6.3: Schematic

d_1 (m)	d_2 (m)	M_2 (kg)	F_2 (N)	θ (N•m)

- By moving the pulley, you can adjust the angle of the force, $\mathbf{F_1}$. Note that when you move the pulley, you also need to move the Spring Scale in order to keep the thread perfectly vertical between the Spring Scale and the pulley.

3. Set the angle of $\mathbf{F_1}$ to each of the values shown in the table below. At each angle, move the Spring Scale toward or away from the pulley as needed so that the magnitude of $\mathbf{F_1}$ is sufficient to balance the beam. Record the force reading on the Spring Scale in newtons.

Angle	F_1 (N)	$\tau_1 = F_1 d_1 \sin\theta$	$(\tau_1 - \tau_2) \div (\tau_1 + \tau_2)/2$
30°			
40°			
50°			
60°			
70°			
80°			

4. Perform the calculations to determine the torque, τ_1, provided by the Spring Scale, and the percent difference between τ_1 and τ_2. [The percent difference is the difference of the two torques divided by the average of the two torques.]

- To provide a consistent mathematical definition of torque, τ_1 and τ_2 must be determined according to the same formula.

5. Apply the generalized definition of torque ($\tau = F\ d \sin\theta$) to the calculation of τ_2 in step 2 previously. [Hint: What is the angle between the force, $\mathbf{F_2}$, and the lever arm, d_2?] Does the calculated value using the generalized definition change the results?

Analyzing Non-Parallel Forces

Imagine two non-parallel forces acting on an object at different distances from its pivot point. The figure shows a diagram for a calculation of torque provided by two non-parallel forces. The force $\mathbf{F_1}$ produces a torque τ_1 about the point O with a magnitude of $F_1\ d_1 \sin\theta_1$. The force $\mathbf{F_2}$ produces a torque τ_2 about the point O with a magnitude of $F_2\ d_2 \sin\theta_2$. However, it would be misleading to simply add the two torques together to determine the total torque because τ_1 and τ_2 cause rotation about point O in opposite directions. When adding two or more torques, add together the magnitudes of all the torques that tend to cause clockwise rotation, then add together the magnitudes of all the torques that tend to cause counterclockwise rotation. For the system to be balanced, the sum of the clockwise torques must equal the sum of the counterclockwise torques.

Remember that the perpendicular distance, $d_\perp$, from the pivot point to the force is equal to $d \sin\theta$, and that the angle is measured between the lever arm, d, and the line of force.

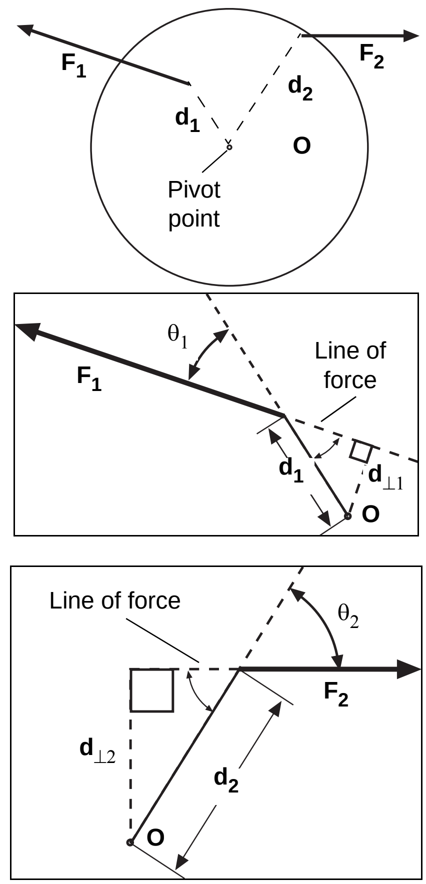


Figure 6.4: Non-Parallel Forces

Torque Wheel

A Torque Wheel provides an easy method for creating an equilibrium among several non parallel forces. Figure 6.5 shows a force **F** applied at an angle θ to the line from the center of the Torque Wheel to the point of application of the force. The torque can be calculated as $\tau = F\ d \sin\theta$. However, as shown, $d \sin\theta$ is just the perpendicular distance, $d_\perp$, between the center of the Torque Wheel and the line of force, when that line is extended far enough.

Imagine a Torque Wheel with two non-parallel forces. The angle between the force $\mathbf{F_1}$ and the lever arm d_1 is θ. The perpendicular distance $d_{1\perp}$ from the pivot point O to the line of force is $d_1 \sin\theta$. Therefore, the torque τ_1 produced by F_1 is $F_1\ d_{1\perp}$.

The radial scale on the Torque Wheel label allows you to measure the perpendicular distance from the pivot point to the line of force. Each concentric circle on the label is 2 mm larger in radius. Each Torque Indicator Arm is transparent and has a centerline that shows the line of force.

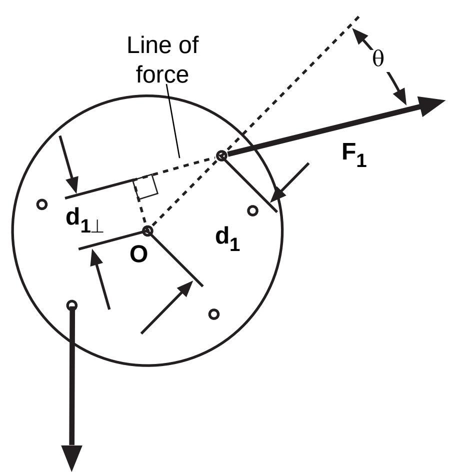


Figure 6.5: Using the Torque Wheel

Set Up the Torque Wheel

Remove the Balance Arm and set up the Torque Wheel on the Statics Board as shown in the diagram. Use pulleys, thread, and hanging masses to apply three torques to the wheel.

Use the radial scale on the Torque Wheel label to measure the perpendicular distance from each line of force to the pivot point. Record the distances in the data table. Calculate and record the forces. Calculate and record the torque for each force using $\tau = F\, d_{\perp}$. Be sure to indicate whether the torque is clockwise or counterclockwise.

Subtract the sum of the clockwise torques from the sum of the counterclockwise torques to determine the total net torque.

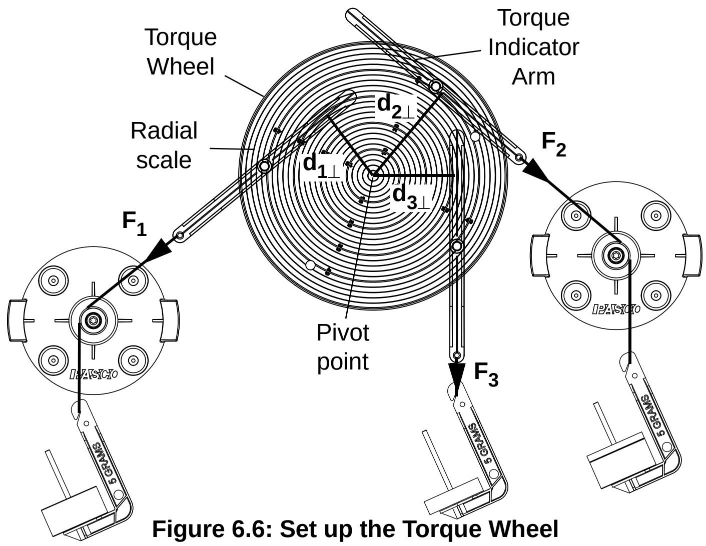


Figure 6.6: Set up the Torque Wheel

Data Table

	Force, F (N)	Perpendicular Distance, $d_{\perp}$ (m)	Torque, $\tau = F\, d_{\perp}$ (N•m)
1			
2			
3			

Total Torque = ________________

Question

Within the limits of your experimental error, is the total net torque equal to zero when the Torque Wheel is in equilibrium.

Extension

Repeat the procedure with different forces and angles.

Exp. 7: The Inclined Plane

Equipment Needed

Item	Item
Statics Board	Inclined Plane and Cart
Pulley (2)	Mounted Spring Scale
Mass and Hanger Set	Thread

Introduction

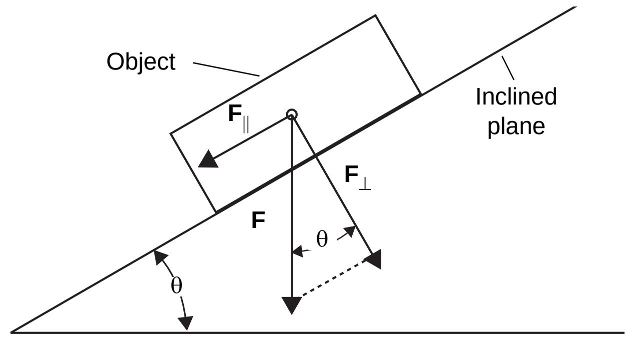


Figure 7.1: The Inclined Plane

Suppose you must design a ramp with a cable to hold a heavy object on an inclined ramp. For a given angle of inclination of the ramp, how much force must the cable deliver to hold the object on the ramp? How much force must the ramp be able to support?

You could solve this problem by building ramps and cable and testing them, or by testing scale models. Alternatively, you could use your knowledge of forces and vectors to solve the problem mathematically. In the diagram, for example, the weight, **F**, of the object on the inclined plane can be resolved into two components: one perpendicular to the plane, $F_{\perp}$, and one parallel to the plane, $F_{\parallel}$. The angle, θ, is the angle of inclination of the inclined plane. In this experiment, you will compare the mathematical solution with data taken directly from a scale model.

Experiment

1. Add a 100-g mass to the cart and measure and record the total mass of the cart. Calculate and record the weight of the cart-plus-mass.

- total mass of cart = _______________ weight of cart = _________________

2. Set up the Inclined Plane on the Statics Board. Start with the plane at 15°. Put the cart on the Inclined Plane and use thread connected under a pulley to the Spring Scale to hold the cart in place on the ramp.

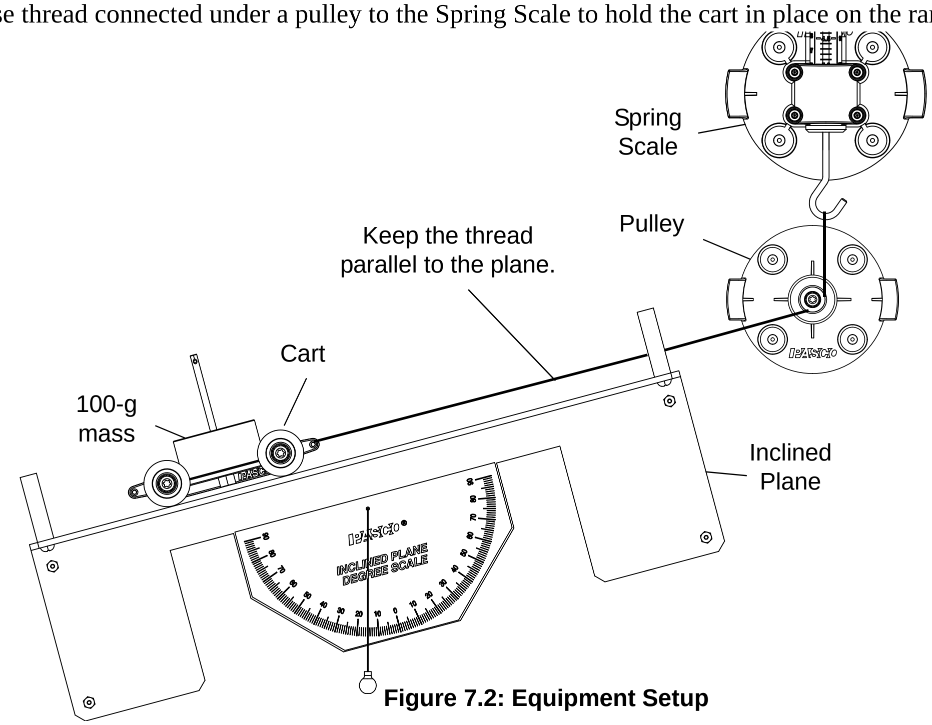


Figure 7.2: Equipment Setup

The force provided by the Spring Scale, $F_{\| \text{ measured}}$, equals the component of the force of gravity that is parallel to the Inclined Plane, $F_{\|}$. The calculated component of force that is parallel to the Inclined Plane, $F_{\| \text{ calculated}}$, is F sin θ, where θ is the angle of the plane.

3. Adjust the angle of inclination of the Inclined Plane to each of the values shown in the table. For accurate values, adjust the pulley and Spring Scale so that the thread remains parallel to the plane. At each value, record the measured value, $F_{\| \text{ measured}}$, of the force parallel to the plane.

4. At each value, calculate the magnitude of the force parallel to the plane, $F_{\| \text{ calculated}} = F \sin \theta$, and record the calculated value.

5. Calculate the percent difference between the measured and calculated values of the force parallel to the plane.

Angle	$F_{\| \text{ measured}}$	$F_{\| \text{ calculated}} = F \sin \theta$	Percent Difference*
15°			
30°			
45°			
60°			
75°			

**The Percent Difference is the absolulte value of the ratio of the difference of the measured and calculated values, divided by the average of the measured plus calculated values, converted to a percentage.*

$$|(\text{measured - calculated}) / (\text{measured + calculated}) / 2| \text{ x } 100\%$$

Question

How well does the calculated force based on the vector model compare to your measured force?

Normal Force

The force that the Inclined Plane provides to support the cart is called the normal force (a force perpendicular – “normal” – to the surface.) In the vector model of the force, the component of force that is perpendicular to the plane, $F_{\perp}$, is F cos θ. The normal force is equal to the force of the cart on the Inclined Plane, perpendicular to its surface.

To measure the force of the cart on the Inclined Plane, reset the Inclined Plane to 15°. Replace the Spring Scale with a mass hanger connected by a thread over the pulley to the cart. Add masses to the mass hanger until the cart and the hanging mass are in equilibrium. (In other words, the force provided by the tension in the thread equals the component of the cart’s weight that is parallel to the plane.)

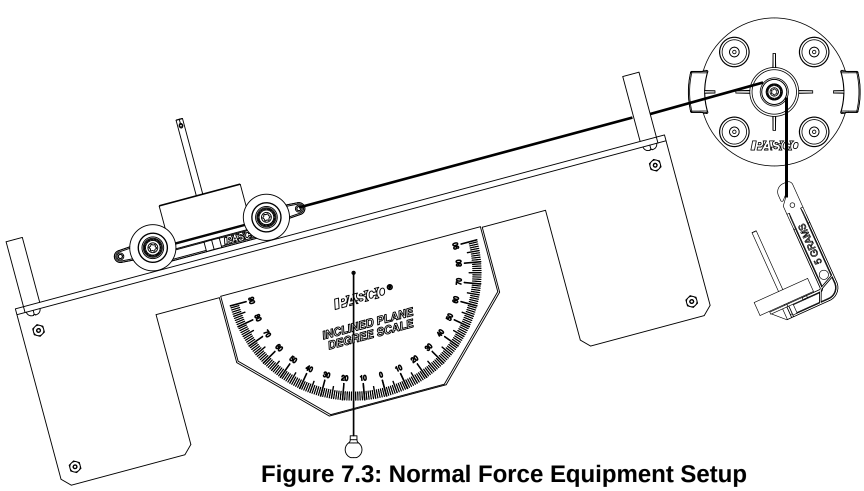


Figure 7.3: Normal Force Equipment Setup

Record the total mass of the mass hanger and calculate and record the weight.

- total mass of mass hanger = ________________ weight of mass hanger = __________________
- How does the weight of the mass hanger compare to value of $F_{\parallel\ calculated}$, the component of the cart's weight that is parallel to the plane? (See the data table for the value at 15°.)

Add a second pulley to the Statics Board and set up the Spring Balance above the pulley. Tie a thread to the hole at the top of the post of the cart. Arrange the pulley so the thread from the post is in line with the post, and therefore is perpendicular to the plane. Arrange the Spring Scale so that the angle of the thread from the pulley up to the scale is vertical.

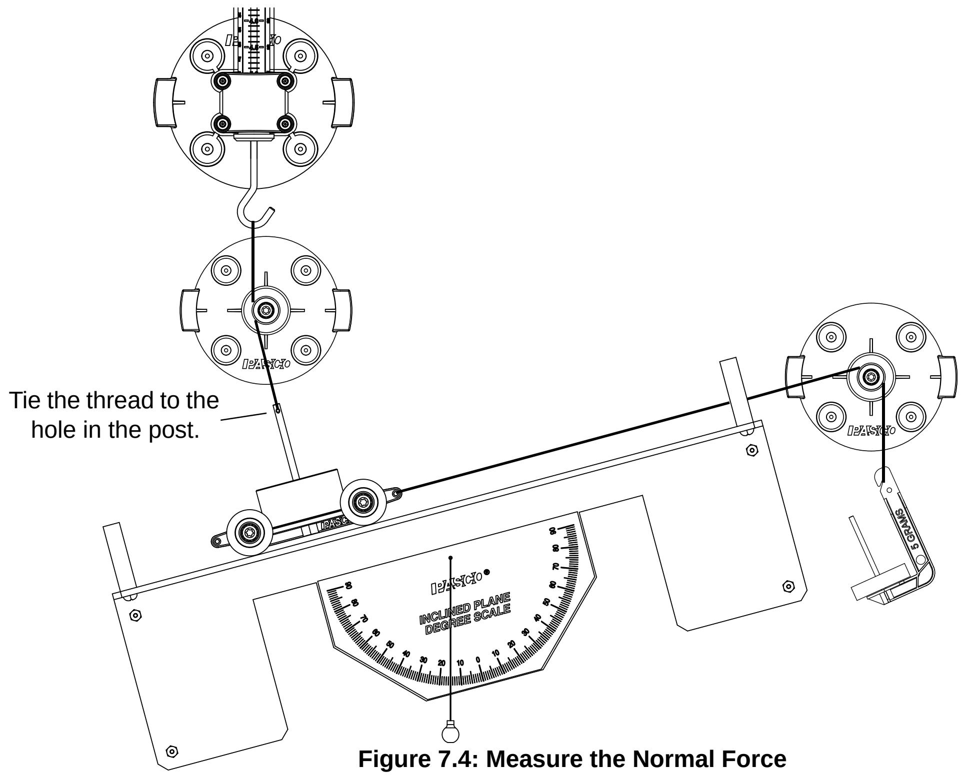


Figure 7.4: Measure the Normal Force

Pull the Spring Scale up until the force just barely lifts the mass cart off the Inclined Plane.

Record the value of the Spring Scale as the perpendicular force, $F_{\perp}$.

$F_{\perp measured}$ = ____________________

Calculate the value of the perpendicular force as predicted by the vector model, $F_{\perp calculated} = F \cos \theta$. Record the value.

$F_{\perp calculated}$ = ____________________

Question

How well does the calculated force based on the vector model compare to your measured force?

Exp. 8: Sliding Friction

Equipment Needed

Item	Item
Statics Board	Inclined Plane and Friction Block
Pulley	Mounted Spring Scale
Mass and Hanger Set	Thread

Theory

Friction is a force between two objects that resists motion between the two objects. Static friction (or sticking friction) is friction between objects that are not moving relative to each other. For example, static friction can prevent an object from sliding down an inclined plane. If a net external force on an object is greater than the force of static friction, the object begins to slide. Sliding friction (or kinetic friction) occurs when two objects are moving relative to each other. In most cases, the static friction between two objects is greater than the sliding friction between the two objects. Once sliding begins, if the sliding object's motion is constant, then the external force equals the force of sliding friction.

An explanation of friction assumes that surfaces are atomically close to each other over a small fraction of their overall area. The surfaces that are atomically close to each other will exert retarding forces on each other. This contact area is proportional to the normal force, and therefore the friction force is proportional to the normal force, or $f \alpha F_N$ where f is the friction force and F_N is the normal force. The ratio of the friction force to the normal force is called the coefficient of friction, μ. The friction force is $f = \mu F_N$. The coefficient of friction is a unitless number that is between zero and one for several common surface-to-surface combinations and is determined empirically.

In this experiment you will investigate how the normal force, the contact materials, and the contact area affect the sliding friction.

Procedure

1. Use the Spring Scale to measure the weight of the Friction Block and then record the value.

- weight, **W**, of friction block = ________________

2. Mount the Inclined Plane on the Statics Board and use the plumb bob to level the plane. Set the Friction Block on the Inclined Plane and use thread to connect it over a pulley to a mass hanger as shown.

3. Adjust the pulley so that the thread is parallel to the Inclined Plane.

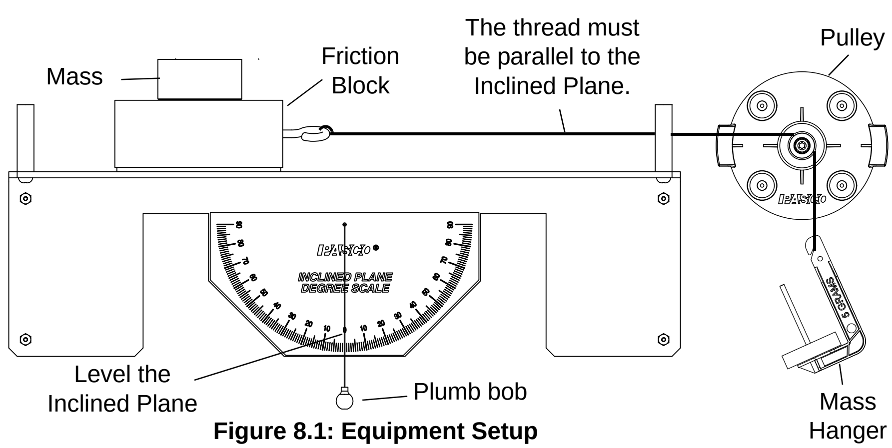


Figure 8.1: Equipment Setup

4. Add or subtract masses on the mass hanger until the Friction Block moves at a very slow, *constant speed* when you give it a small push.

- If the Friction Block stops, the mass is too light. If the Friction Block accelerates, the mass is too heavy.
- The weight of the hanging mass that is sufficient to pull the Friction Block at a *constant speed* is $\boldsymbol{f_k}$, the force of the sliding (kinetic) friction between the Friction Block and the Inclined Plane.

Variables

Change the following factors and measure the sliding friction force.

- **Normal Force**: Add mass to the top of the Friction Block to increase the normal force between the block and the Inclined Plane.
- **Contact Material**: Two sides of the Friction Block are bare wood. Two other sides are covered with felt. Compare the sliding friction force for a wood surface to the sliding friction force for an equally sized felt surface.
- **Contact Area**: The top and bottom surfaces have a larger area than the side surfaces. Compare the sliding friction force for a larger area to the sliding friction force for a smaller area.

Reminders for each trial:

- Carefully adjust the mass on the mass hanger until the weight of the hanging mass is enough so that the Friction Block moves at a very slow, constant speed after you give it a small push.
- Record the total mass of the Friction Block, M, and the hanging mass, m, (total mass of the mass hanger and added masses).
- Calculate and record the normal force, $\mathbf{F_N}$, of the Friction Block on the Inclined Plane and the sliding friction force, $\mathbf{f}_k$ (weight of the hanging mass).
- Calculate and record the coefficient of friction, μ, which is the ratio of the sliding friction force, $\mathbf{f_k}$, divided by the normal force, $\mathbf{F_N}$

Data Table

Trial	Added Mass (kg)	Total Mass, M, of Block	Normal Force, F_N = Mg	Hanging Mass, m	Friction force, f_k = mg	Coefficient of friction, $\mu = f_k/F_N$
1	0.000					
2	0.050					
3	0.100					
4	0.150					
5	0.200					
6	0.250					
Trial	**Surface**	**Total Mass, M, of Block***	**Normal Force, F_N = Mg**	**Hanging Mass, m**	**Friction force, f_k = mg**	**Coefficient of friction, $\mu_k = f_k/F_N$**
7	Wood, Large					
8	Felt, Large					
9	Wood, Small					
10	Felt, Small					

*For trials 7 through 10, let the total mass of the Friction Block be constant.

Questions

1. In trials 1 through 6, what happens to the sliding friction as the normal force increases?

2. In trials 1 through 6, what happens to the coefficient of friction as the normal force increases?

3. How does the sliding friction for the large wood surface compare to the sliding friction for the large felt surface? How does the sliding friction for the small wood surface compare to the sliding friction for the small felt surface?

4. Based on your measurements, does the sliding friction between two objects depend on the materials that are in contact?

5. How does the sliding friction for the large wood surface compare to the sliding friction for the small wood surface? How does the sliding friction for the large felt surface compare to the sliding friction for the small felt surface?

6. Based on your measurements, does the sliding friction between two objects depend on the area of contact between the objects?

Sliding Friction on an Inclined Plane

If the Friction Block is not on a horizontal surface, will the coefficient of sliding friction, μ_k, be different?

When an object is on the Inclined Plane at an angle, one component of the object's weight (**F** = m**g**) is parallel to the surface of the plane ($F_{||}$), and another component of F is perpendicular to the plane ($F_{\perp}$). In theory, this perpendicular component is equal to the normal force of the surface of the plane.

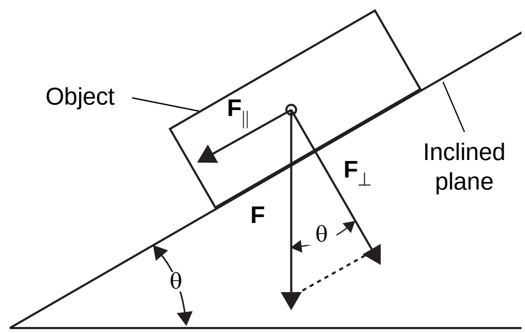


Figure 8.2: The Inclined Plane

If there was no friction between the object and the plane, the parallel component of the object's weight would accelerate the object down the plane. However, because there is friction between the object and the plane, the force of friction ($f = \mu F_N$) opposes the parallel component of force. In other words, the vector for the force of friction would point *up* the plane as the object slides *down* the plane.

Imagine that the object is being pulled up the plane by the tension of a thread connected to a hanging mass. Would the sliding friction ($f_k = \mu_k F_N$) oppose the parallel component of the object's weight, or would the vector for the sliding friction be in the same direction as the parallel component? Both forces would point *down* the plane as the object is pulled *up* the plane. If the object is pulled at a constant speed, then the net force on the object is zero. Would the tension in the thread, T, equal the sum of the parallel component and the sliding friction?

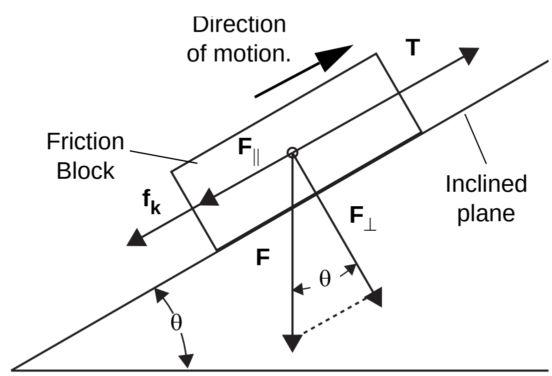


Figure 8.3: Force Diagram

Prediction

How will the sum of the parallel component of the block's weight ($F_{||}$) plus the sliding friction (f_k) compare to the weight of the hanging mass?

Procedure

1. Measure and record the weight, W, of the Friction Block.

- weight, W =__________

2. Mount the Inclined Plane on the Statics Board and set the angle, θ, to 15°. Set up the pulley, mass hanger, thread, and Friction Block as shown in the figure. Make sure that the thread is parallel to the Inclined Plane.

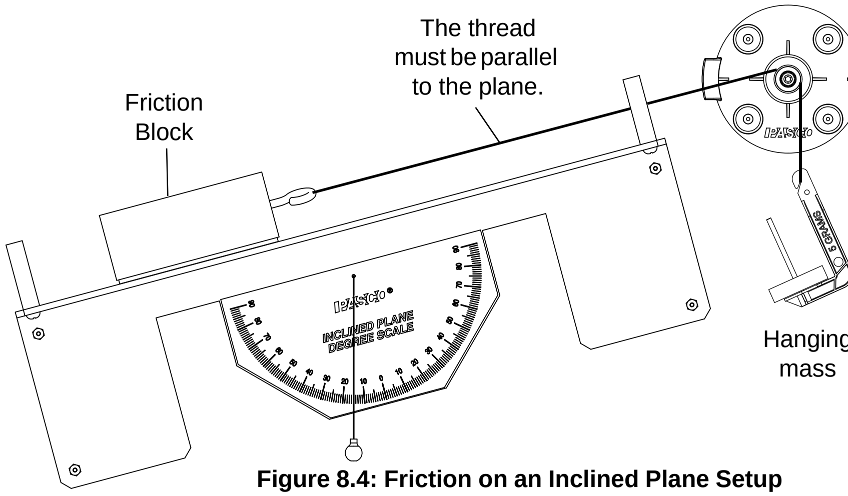


Figure 8.4: Friction on an Inclined Plane Setup

3. Carefully adjust the mass on the mass hanger until the weight of the hanging mass is enough so that the Friction Block moves at a very slow, constant speed up the inclined after you give it a small push.

4. Record the total mass of the hanging mass, m, (mass of the mass hanger and added masses).

Calculations

- Remember that the weight of the Friction Block, W = $\mathbf{F}$ = M$\mathbf{g}$.

5. Calculate and record the parallel component, $\mathbf{F}_{\|}$ = M$\mathbf{g}$ sin θ, of the block's weight.

6. Calculate the perpendicular component, $\mathbf{F}_{\perp}$ = M$\mathbf{g}$ cos θ, of the block's weight and record this as the normal force, $\mathbf{F_N}$.

7. Use the normal force, $\mathbf{F_N}$, and the coefficient of sliding friction, $\mu_\mathbf{k}$, for the surface material of the block (either wood or felt) to calculate the sliding friction force, $\mathbf{f_k} = \mu_\mathbf{k}\mathbf{F_N}$.

8. Calculate the weight of the hanging mass, $\mathbf{F_{hanging}}$ = m$\mathbf{g}$ and record this as the tension in the thread, $\mathbf{T}$.

9. Calculate and record the sum of the parallel component of the block's weight, $\mathbf{F}_{\|}$, and the sliding friction force, $\mathbf{f_k}$.

Data Table

Angle, θ	Block Mass, M	Hanging Mass, m	Parallel Component, $F_{\|}$ = Mg sin θ	Normal Force, $F_{\perp} = F_N$ = Mg cos θ	Friction Force, $f_k = \mu_k F_N$	Tension, T = mg

- Sum of the parallel component of the block's weight, $\mathbf{F}_{\|}$, and the sliding friction force, $\mathbf{f_k}$ = ______________.

Question

How does the tension in the thread compare to the sum of the parallel component of the block's weight and the sliding friction force?

Static Friction on an Inclined Plane

Imagine that the Friction Block is placed on the Inclined Plane, and one end of the plane is tilted upward until the parallel component of the block's weight begins to pull the block down the plane. Static friction holds the block in place on the plane until the parallel component of the block's weight is larger than the static friction. If the block will almost — but not quite — start sliding, then the static friction is as large as possible and equals the parallel component of the block's weight.

One way to measure the coefficient of static friction, μ_k, is to find the maximum angle at which the plane can be tilted before the block begins sliding down the plane. The component of the weight of the block that is parallel to the plane, $\mathbf{F}_{||}$, is M**g** sin θ, where M is the mass of the block. The normal force, $\mathbf{F_N}$, is the perpendicular component of the block's weight, or $\mathbf{F}_{\perp}$ = M**g** cos θ. The force of static friction, $\mathbf{f_s}$, is the coefficient of static friction, μ_s, multiplied by the normal force. Therefore, $\mathbf{f_s} = \mu_s$ M**g** cos θ.

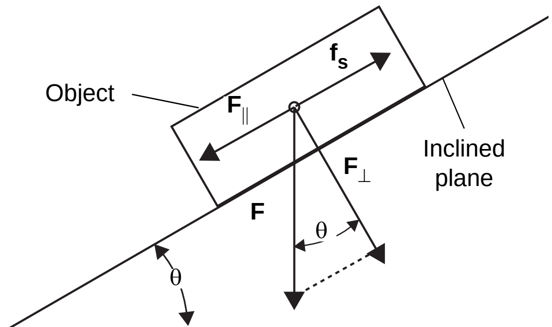


Figure 8.5: Static Friction Diagram

When the block is at rest, the static friction force equals the component of the block's weight that is parallel to the incline, or $Mg\sin\theta = \mu_s Mg\cos\theta$. Solving for the coefficient of static friction, μ_s, gives the following:

$$\mu_s = \frac{Mg\sin\theta}{Mg\cos\theta} = \tan\theta$$

Procedure

1. Mount the Inclined Plane on the Statics Board and set the angle to zero. Place the Friction Block on the Inclined Plane.

2. Carefully raise one end of the Inclined Plane until the Friction Block just begins to slide. Record the angle, θ.

3. Switch the Friction Block to a different surface material and repeat the procedure.

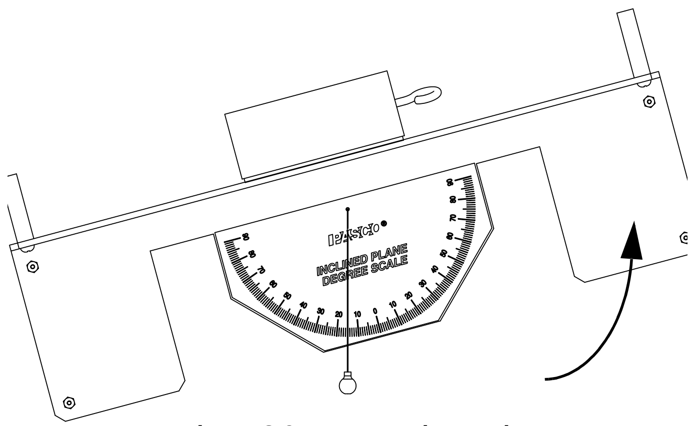


Figure 8.6: Increase the Angle

Data Table

Surface Material	Angle	Coefficient of Static Friction
Wood		
Felt		

Calculations

- Calculate the coefficient of static friction for both wood and felt.

Question

- How does the coefficient of static friction for wood compare to the coefficient of static friction for felt?

Exp. 9: Simple Harmonic Motion–Mass on a Spring

Equipment Needed

Item	Item
Statics Board	Mounted Spring Scale
Mass and Hanger Set	Thread
Stopwatch (ME-1234)	

Theory

Imagine a mass hanging from a spring. At rest, the mass hangs in a position such that the spring force just balances the gravitational force on the mass (its weight). When the mass is pulled below this original point (called the equilibrium position), the spring exerts a force to pull it back up. When the mass is above this original point, gravity pulls it down. The net force on the mass is therefore a restoring force because it always acts to accelerate the mass back toward its equilibrium position.

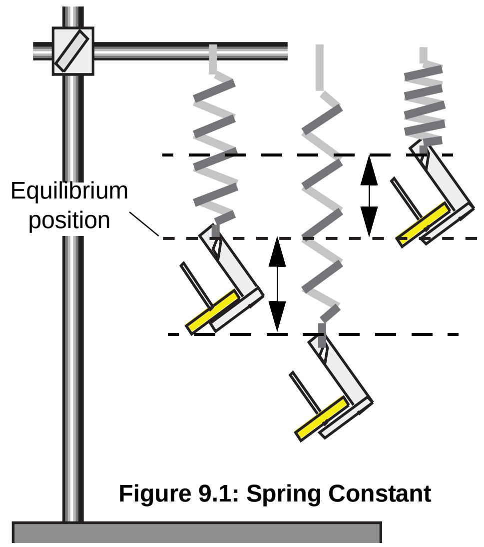


Figure 9.1: Spring Constant

Previously you may have investigated Hooke's Law, which states that the force exerted by a spring is proportional to the distance beyond its normal length to which it is stretched. (This also is true for the compression of a spring.) This relationship is stated as $\boldsymbol{F} = -k\boldsymbol{x}$, where $\boldsymbol{F}$ is the force exerted by the spring, $\boldsymbol{x}$ is the displacement of the end of the spring from the equilibrium position, and k is the constant of proportionality, called the spring constant.

Whenever an object is acted on by a restoring force that is proportional to the displacement of the object from its equilibrium position, the resulting motion is called Simple Harmonic Motion (SHM). When the simple harmonic motion of a mass, M, on a spring is analyzed mathematically using Newton's Second Law (and calculus), the period of the motion, T, is as follows:

$$T = 2\pi\sqrt{\frac{M}{k}}$$

The period, T, is the amount of time for one complete oscillation (down-up-down). In this experiment you will investigate this equation for the period of simple harmonic motion.

Procedure

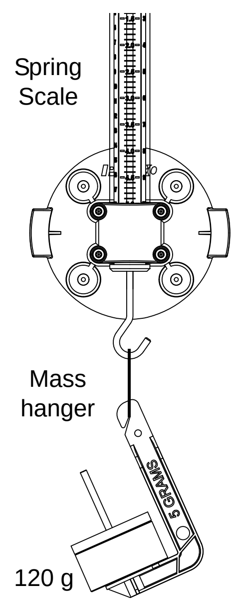


Figure 9.2: Setup

1. Measure and record k, the spring constant for the spring in the Spring Scale (see Exp: Hooke's Law).

- spring constant, k = _________________ (N/m)

2. Mount the Spring Scale on the Statics Board so that the scale is perfectly vertical. Use thread to hang a mass hanger from the scale, and add 120 g of mass to the hanger (for a total hanging mass of 125 g (0.125 kg).

- Practice the following: Pull the mass hanger down several centimeters and release it smoothly so that the mass hanger oscillates up and down without moving from side to side.

3. Pull the mass hanger down and release it smoothly. Let it oscillate a few times before taking any measurements. When the oscillations are smooth and regular, measure the time for at least ten complete oscillations (down-up-down). If possible, measure the time for as many oscillations as you can before the amplitude of the oscillations becomes too small. Record the number of oscillations and the total time.

4. Calculate and record the Measured Period, $T_{measured}$, by dividing the total time by the number of oscillations.

5. Repeat the measurement for the 125 g mass five times. Calculate and record the Average Measured Period.

6. Use M and k to calculate the Theoretical Period, $T_{theoretical}$, and record the result.

7. Repeat the procedure using masses of 175 g and 225 g (total mass including the mass hanger).

Data Table

Mass (kg)	Oscillations	Total Time (s)	Measured Period (s)	Theoretical Period (s)
0.125 kg				
0.125 kg				
0.125 kg				
0.125 kg				
0.125 kg				
		Average Measured Period (s)		
0.175 kg				
0.175 kg				
0.175 kg				
0.175 kg				
0.175 kg				
		Average Measured Period (s)		
0.225 kg				
0.225 kg				
0.225 kg				
0.225 kg				
0.225 kg				
		Average Measured Period (s)		

Questions

1. How well does the theoretical value for the period of oscillation compare to the measured period of oscillation?

2. Does the equation for the period of a mass on a spring provide a good mathematical model for the physical reality? Why or why not?

$$T = 2\pi\sqrt{\frac{M}{k}}$$

Extension

In addition to the hanging mass, there is other mass that is oscillating up and down. The rod inside the Spring Scale moves up and down as well. A portion of the spring itself moves as the hanging mass oscillates. Would this unaccounted for mass make a difference in your calculation of the theoretical period, $T_{theoretical}$?

To find out, used the equation for the period of oscillation to calculate what the total oscillating mass, M_{total}, should be based on the spring constant and the Average Measured Period, $T_{measured}$, for each mass. Use the formula,

$$M_{total} = \frac{T^2 k}{4\pi^2}$$

where *T* is the Average Measured Period and *k* is the spring constant.

Hanging Mass (kg)	Average Measured Period (s)	Calculated Total Mass (kg)	New Theoretical Period (s)
0.125 kg			
0.175 kg			
0.225 kg			

After calculating the total mass based on the average measured period and the spring constant, recalculate the theoretical period based on the calculated total mass and the spring constant:

$$T_{new} = 2\pi\sqrt{\frac{M_{total}}{k}}$$

Compare the calculated total mass, M_{total}, to the hanging mass, M. Approximately how much "extra" mass is oscillating in addition to the hanging mass?

- M_{extra} = ______________

Question

1. For each trial, how well does the *new* theoretical value for the period of oscillation compare to the average measured period of oscillation?

2. Does the equation for the period of an oscillating mass on a spring provide a good mathematical model for the physical reality? Why or why not?

Simple Harmonic Motion–Beam on a Spring

Imagine a horizontal beam that is supported by a hinge at one end and a vertical spring at the other end. If the end of the beam is pulled down, the spring exerts a restoring force, $\boldsymbol{F} = -k\boldsymbol{x}$, to return the beam to its equilibrium position. The beam will oscillate up and down with a period, T_{beam}. For a *mass* on a spring, the period, T, is as follows:

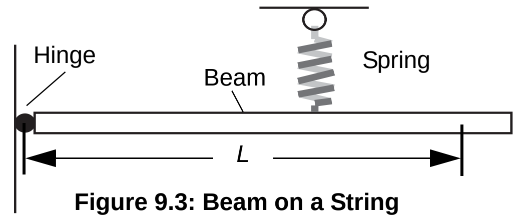


Figure 9.3: Beam on a String

$$T = \sqrt{\frac{M}{k}}$$

where M is the total oscillating mass and k is the spring constant. What is the period for a *beam* on a spring?

The beam rotates about the hinge as the end attached to the spring oscillates. The force of the spring on the oscillating end of the beam, $\boldsymbol{F} = -k\boldsymbol{x}$, produces a torque on the beam. Let L be the length of the lever arm of the beam. The torque due to the spring is $\tau = \boldsymbol{F}L$. A net torque causes angular acceleration, α, that is directly proportional to the torque, τ, and inversely proportional to the rotational inertia, I. That is,0,

$$\alpha = \frac{\tau}{I}$$

or $\tau = \alpha I$. Setting the two expressions for torque equal to each other gives $\boldsymbol{F}L = \alpha I$. Assume that the beam is like a thin rod pivoted around one end. The rotational inertia of the thin rod is $I = 1/3\ m\ L^2$ where m is the mass of the beam. Since the force on the beam is $\boldsymbol{F} = -k\boldsymbol{x}$, the equation $\boldsymbol{F}L = \alpha I$ becomes:

$$-kxL = \frac{\alpha m L^2}{3}$$

The angular acceleration, α, and the tangential (linear) acceleration, a_T, of the oscillating end of the beam are related. The tangential acceleration, $a_T = \alpha r$ where r is the radius of rotation. In this case, the radius of rotation is the lever arm, L, so $a_T = \alpha L$, *or* $\alpha = a_T/L$. The expression becomes:

$$-kxL = \frac{a_T m L^2}{3L}$$

which simplifies to

$$-kx = \frac{a_T m}{3}$$

Solving for the tangential acceleration gives:

$$a_T = -\frac{3k}{m}x$$

The tangential acceleration, $a_T = -\omega^2 x$, so the expression becomes:

$$-\omega^2 x = -\frac{3k}{m}x$$

$$\omega^2 = \frac{3k}{m}$$

$$\omega = \sqrt{\frac{3k}{m}}$$

Since the angular frequency, $\omega = 2\pi/T$, the period, $T = 2\pi/\omega$, or:

$$T = 2\pi\sqrt{\frac{m}{3k}}$$

In this part of the experiment you will investigate this equation for the simple harmonic motion of a beam on a spring.

Procedure

1. Mount a Protractor on one end of the Balance Arm. Measure and record the total mass of the arm plus protractor.

- mass, *m* = ____________

2. Add the Pivot to the other end of the Balance Arm and mount the Balance Arm on the Statics Board.

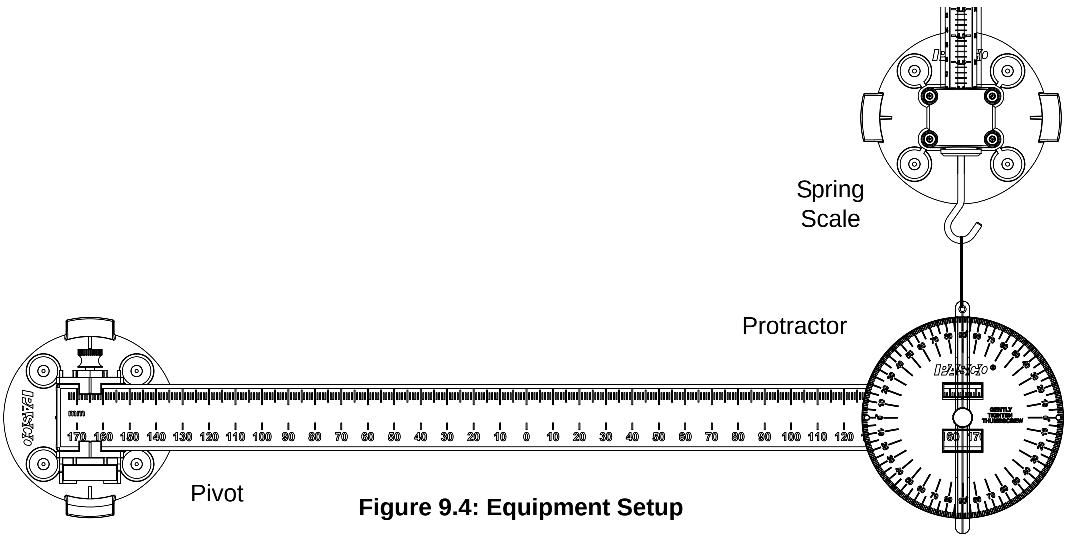


Figure 9.4: Equipment Setup

3. Mount the Spring Scale on the Statics Board so that the scale is perfectly vertical. Use thread to connect the Angle Indicator on the Protractor to the bottom hook of the Spring Scale.

4. Adjust the Balance Arm and the Spring Scale so that the arm is horizontal and the scale is vertical and directly above the center of the protractor.

5. Push the balance arm down and release it smoothly. Measure the time for at least ten complete oscillations (down-up-down). Record the number of oscillations and the total time.

6. Calculate and record the Measured Period, $T_{measured}$, by dividing the total time by the number of oscillations.

7. Repeat the measurement five times. Calculate and record the Average Measured Period.

8. Use *m* and *k* to calculate the Theoretical Period, $T_{theoretical}$, and record the result.

Data Table

Trial	Oscillations	Total Time (s)	Measured Period (s)	Theoretical Period (s)
1				
2				
3				
4				
5				
		Average Measured Period (s)		

Questions

1. How well does the theoretical value for the period of oscillation compare to the average measured period of oscillation?

2. Does the equation for the period of a beam on a spring provide a good mathematical model for the physical reality? Why or why not?

Extension

The rod and part of the spring in the Spring Scale oscillate up and down as the beam oscillates. Use your results from the first part of this experiment to calculate the total mass that oscillates: the arm and protractor plus the mass of the parts of the Spring Scale that also oscillate. Recalculate the theoretical period, $T_{theoretical}$.

- How does the re-calculated theoretical period compare to the average measured period?

Exp. 10: Simple Harmonic Motion–The Simple Pendulum

Equipment Needed

Item	Item
Statics Board	Utility Mount and Cord Clip
Thread	Mass and Hanger Set
Stopwatch (ME-1234)	

Theory

Simple harmonic motion is not limited to masses on springs. In fact, it is one of the most common and important types of motion found in nature. From the vibrations of atoms to the vibrations of airplane wings, simple harmonic motion plays an important role in many physical phenomena.

A swinging pendulum, for example, shows behavior that is very similar to that of a mass on a spring. By making comparisons between these two phenomena, some predictions can be made about the period of oscillation for a pendulum.

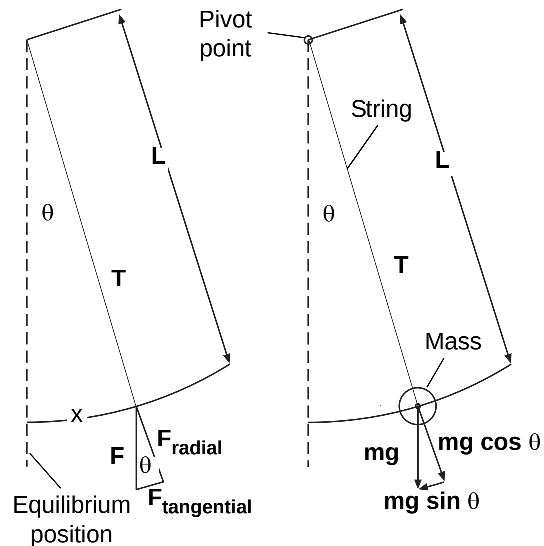


Figure 10.1: Pendulum

The figure shows a simple pendulum with a string and a mass at an angle θ from the vertical position. Two forces act on the mass: the force of the string, **T**, and the force of gravity. The gravitational force, **F** = m**g**, can be resolved into two components. One component, $\mathbf{F}_{\mathbf{radial}}$, is along the string. The other component, $\mathbf{F}_{\mathbf{tangential}}$, is perpendicular to the string and tangent to the arc of the mass as it swings. The radial component of the weight, m**g** cos θ, equals the force, **T**, through the string. The tangential component of the weight, m**g** sin θ, is in the direction of motion and accelerates or decelerates the mass.

Using the congruent triangles in the figure, it can be seen that the displacement of the mass from the equilibrium position is an arc whose length, x, is approximately L tan θ. If the angle, θ, is relatively small (less than 20°), then it is very nearly true that sin θ = tan θ. Therefore, for small swings of the pendulum, it is approximately true that $\mathbf{F}_{\mathbf{tangential}}$ = m**g** tan θ = m**g** x/L. Since the tangential force is a restoring force, the equation should be $\mathbf{F}_{\mathbf{tangential}}$ = - m**g** x/L. Comparing this equation to the equation for the restoring force of a mass on a spring, **F** = -k**x,** it can be seen that the quantity mg/L has the same mathematical role as k, the spring constant. On the basis of this similarity, you can say that the period of oscillation for a pendulum is as follows:

$$T = 2\pi\sqrt{\frac{m}{\frac{mg}{L}}} = 2\pi\sqrt{\frac{L}{g}}$$

where *m* is the mass, *g* is the acceleration due to gravity, and *L* is the length of the pendulum from the pivot point to the center of mass of the hanging mass.

In this part of the experiment you will investigate this equation for the period of the simple harmonic motion of a pendulum.

Procedure

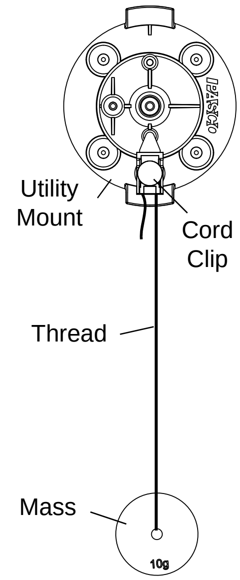


Fig. 10.2: Setup

1. Place the Utility Mount near the top edge of the Statics Board. Loop a thread approximately 45 cm long through a Cord Clip and attach the Cord Clip to the mount.

2. Attach a 10 g mass to the thread and adjust the length of the thread so that the pendulum is as long as possible on the board.

3. Measure and record *L*, the length of the pendulum from the pivot point to the center of mass of the hanging mass. Record *m*, the hanging mass.

4. Set the mass swinging but keep the angle of the swing reasonably small (less than 20°). Measure the time for 30 oscillations. Record the total time.

5. Repeat the measurement five times.

6. Change the mass. Repeat the procedure for a 20 g hanging mass and then a 50 g hanging mass.

7. Change the length. Repeat the procedure for the original mass and two different pendulum lengths; one-half of the original length and then one-quarter of the original length.

Data Table

Mass 1 (kg)	Length 1 (m)	Oscillations	Total Time (s)	Measured Period (s)	Theoretical Period (s)
0.010					
Average Measured Period (s)					
Mass 2 (kg)	**Length 1 (m)**	**Oscillations**	**Total Time (s)**	**Measured Period (s)**	**Theoretical Period (s)**
0.020					
Average Measured Period (s)					
Mass 3 (kg)	**Length 1 (m)**	**Oscillations**	**Total Time (s)**	**Measured Period (s)**	**Theoretical Period (s)**
0.050					
Average Measured Period (s)					

Data Table

Mass 1 (kg)	Length 2 (m)	Oscillations	Total Time (s)	Measured Period (s)	Theoretical Period (s)
0.010					
			Average Measured Period (s)		
Mass 1 (kg)	**Length 3 (m)**	**Oscillations**	**Total Time (s)**	**Measured Period (s)**	**Theoretical Period (s)**
0.010					
			Average Measured Period (s)		

Calculations

1. Calculate and record the Measured Period by dividing the total time by the number of oscillations.

2. Calculate and record the Average Measured Period.

3. Calculate the Theoretical Period.

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Questions

1. How well does the theoretical value for the period of oscillation for a simple pendulum compare to the measured period of oscillation?

2. Does the equation for the period of a simple pendulum provide a good mathematical model for the physical reality? Why or why not?

3. How does increasing the mass of the simple pendulum affect the period of the pendulum?

4. How does changing the length of the simple pendulum affect the period of the pendulum?

Exp. 11A: Simple Harmonic Motion–Physical Pendulum

Equipment Needed

Item	Item
Statics Board	Balance Arm
Stopwatch (ME-1234)	

Theory

A simple pendulum is a mass on the end of a "massless" length of string. A physical pendulum is any rigid body that pivots at some point of the body so that it can rotate freely in a vertical plane under the force of gravity. Let $\mathbf{L_{cm}}$ be the distance from the pivot point to the center of mass of the body.

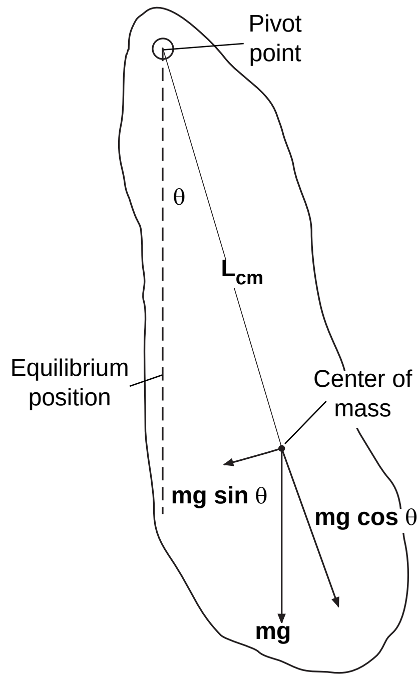


Figure 11.1: Physical Pendulum

The torque that causes the rotation is produced by the component of the force of gravity that is perpendicular to the line that joins the pivot point to the center of mass, $m\mathbf{g} \sin \theta$. The torque is the product of the lever arm, $\mathbf{L_{cm}}$, and the perpendicular force, or $\tau = \mathbf{L_{cm}}\, m\mathbf{g} \sin \theta$. For small angles, $\sin \theta = \theta$, so the expression for the torque becomes $\tau = \mathbf{L_{cm}}\, m\mathbf{g}\, \theta$ or $\tau = k\theta$ where $k = \mathbf{L_{cm}}\, m\mathbf{g}$.

Since the expression for torque matches the expression for the restoring force, $\mathbf{F} = -k\mathbf{x}$, on an object in simple harmonic motion, the period of oscillation for the physical pendulum can be written as follows:

$$T = 2\pi\sqrt{\frac{I}{L_{cm}mg}}$$

where I, the moment of inertia (rotational inertia) for the physical pendulum, replaces M and $\boldsymbol{L_{cm}}\, m\boldsymbol{g}$ replaces k in the equation for the period of oscillation of a mass on a spring,

$$T = 2\pi\sqrt{\frac{M}{k}}$$

Assume that the Balance Arm beam is a rectangular-type rod. For a rod pivoting around an axis through its center of mass, the moment of inertia around the center of mass is:

$$I_{cm} = \frac{1}{12}mL^2$$

where m is the mass and L is the length of the rod.

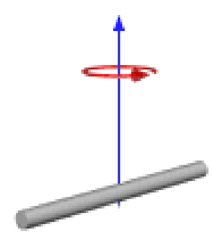


Fig. 11.2: Moment of Inertia

What happens if the rod pivots around one end rather than its center of mass? If the rod pivots around any other axis that is parallel to the axis through the center of mass, you can use the Parallel Axis Theorem to calculate the moment of inertia around that parallel axis. The moment of inertia about the parallel axis, $I_{parallel}$, is the sum of the moment of inertia around the center of mass, I_{cm}, plus mL_{cm}^{2}, where m is the mass of the rod and L_{cm} is the perpendicular distance from the center of mass to the pivot point, or

$$I_{parallel} = I_{cm} + mL_{cm}^2 = \frac{1}{12}mL^2 + mL_{cm}^2$$

The formula for the period of oscillation becomes

$$T = 2\pi\sqrt{\frac{\frac{1}{12}mL^2 + mL_{cm}^2}{L_{cm}mg}} = 2\pi\sqrt{\frac{\frac{1}{12}L^2 + L_{cm}^2}{L_{cm}g}}$$

In this part of the experiment you will investigate this equation for the period of the simple harmonic motion of a physical pendulum with a fixed distance between the pivot point and the center of mass.

Procedure

1. Move the pivot of the Balance Arm to the 170 mm mark at one end of the beam, and mount the pivot near the top of the Statics Board.

2. Measure and record the total length, L, of the beam of the Balance Arm and the distance, L_{cm}, from the pivot point to the center of mass (presumably at “0”).

3. Start the beam swinging but keep the angle of the swing reasonably small (less than 20°).

4. Measure and record the total time for 10 oscillations.

5. Repeat the measurement a total of five times.

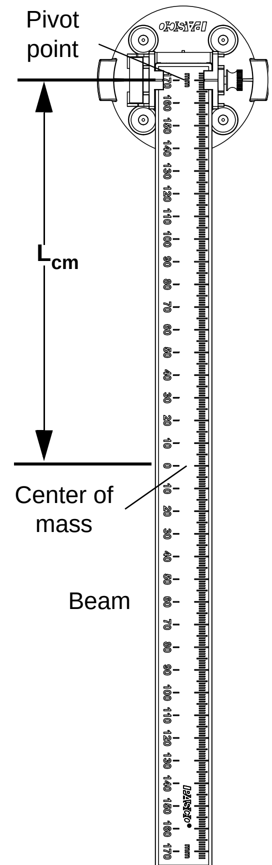


Fig. 11.3: Setup

Data Table

- Length, L (m) = ________________ Distance, L_{cm} (m) = _______________

Trial	Oscillations	Total Time (s)	Measured Period (s)	Theoretical Period (s)
1				
2				
3				
4				
5				
		Average Measured Period (s)		

Calculations

1. For each trial, calculate and record the Measured Period by dividing the total time by the number of oscillations. Calculate the Average Measured Period.

2. Calculate the Theoretical Period:

$$T = 2\pi\sqrt{\frac{\frac{1}{12}L^2 + L_{cm}{}^2}{L_{cm}g}}$$

Questions

1. How well does the theoretical value for the period of oscillation compare to the measured period of oscillation?

2. Does the equation for the theoretical period of this physical pendulum provide a good mathematical model for the physical reality? Why or why not?

Try This

- If you assume that the distance L_{cm} from the pivot point to the center of mass is $L/2$, exactly half the length of the rod, then the formula for $I_{parallel}$ gives a simpler expression:

$$I_{parallel} = I_{cm} + mL_{cm}{}^2 = \frac{1}{12}mL^2 + mL_{cm}{}^2 = \frac{1}{12}mL^2 + m\left(\frac{L}{2}\right)^2 = \frac{1}{12}mL^2 + \frac{1}{4}mL^2 = \frac{1}{3}mL^2$$

- Substituting this expression into the formula for the period of oscillation of a physical pendulum gives:

$$T = 2\pi\sqrt{\frac{I_{parallel}}{L_{cm}mg}} = 2\pi\sqrt{\frac{\frac{1}{3}mL^2}{L_{cm}mg}} = 2\pi\sqrt{\frac{\frac{1}{3}mL^2}{\frac{L}{2}mg}} = 2\pi\sqrt{\frac{2L}{3g}}$$

- Use this expression to re-calculate the theoretical period of the physical pendulum and compare the result to the measured period of oscillation.

- How much difference is there between the re-calculated theoretical period and the previous theoretical period?

Extension: Period of Oscillation for Large Angles

The motion of a physical pendulum is simple harmonic motion for small angles. When the angle becomes larger, $\sin \theta \neq \theta$. How does the period of oscillation for a physical pendulum vary as the initial angle of swing increases?

Equipment Needed

Item	Item
Statics Board	Force Wheel
Balance Arm	Stopwatch (ME-1234)
Pencil or Dry Erase Marker Pen	Thread

Setup

1. Slide the Balance Arm pivot to one end of the beam, and mount the pivot in a top corner of the Statics Board.

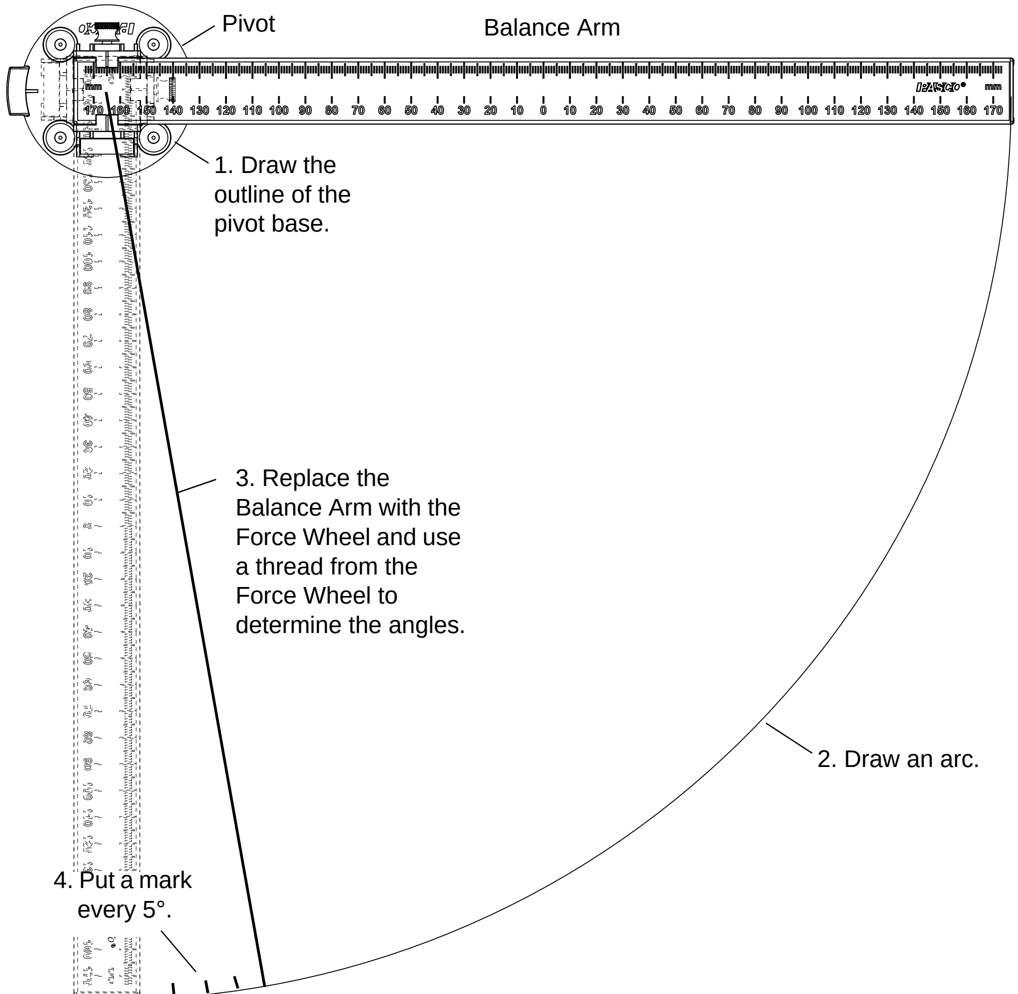


Figure 11.4: Large Angle Oscillation Setup

2. Use a dry erase marker pen or a pencil to draw the outline of the pivot base onto the board.

3. Hold the dry erase pen or pencil at the end of the beam and perpendicular to the board, and move the beam and pen so you draw an arc from the bottom corner of the board to the opposite top corner.

4. After you draw the arc, remove the Balance Arm and place the base of the Force Wheel into the outline of the pivot base. Level the Force Wheel.

5. Stretch one of the threads from the Force Disk to the arc on the board. Arrange the thread so it is straight down from the Force Wheel, and put a mark on the arc and label it as 0°.

6. Keep the thread taut and move it so the angle relative to vertical is 5°. Put a mark on the arc and label it.

7. Continue to move the thread along the arc and put a mark at every 5° interval until the string is horizontal.

8. Remove the Force Wheel and replace the Balance Arm on the board in its original position.

Procedure

1. Hold the beam of the Balance Arm so its center line points at the 5° mark on the arc. Release the beam and measure the time for at least ten oscillations. Record the time and the number of oscillations.

2. Hold the beam so its center line points at the 10° mark and repeat the measurement.

3. Continue to measure and record the total time for at least ten oscillations at each angle until you reach 90°.

Data Table

Angle	Oscillations	Total Time (s)	Period (s)	Angle	Oscillations	Total Time (s)	Period (s)
5				50			
10				55			
15				60			
20				65			
25				70			
30				75			
35				80			
40				85			
45				90			

Analysis

1. Calculate and record the period for each angle by dividing the total time by the number of oscillations.

2. Create a graph of period versus angle.

Question

1. At what angle does the period of the physical pendulum begin to change?

2. Does the period increase or decrease as the initial angle increases?

Exp. 11B: Minimum Period of a Physical Pendulum

Equipment Needed

Item	Item
Statics Board	Balance Arm
Stopwatch (ME-1234)	

Theory

The period of oscillation for a physical pendulum can be written as follows:

$$T = 2\pi\sqrt{\frac{I}{L_{cm}mg}}$$

where I is the moment of inertia (rotational inertia) for the physical pendulum, L_{cm} is the perpendicular distance from the axis at the pivot point to the parallel axis at the center of mass, *cm*, and m is the mass of the pendulum.

The moment of inertia, I_{cm}, for a rectangular-type rod about its center of mass is:

$$I_{cm} = \frac{1}{12}m(a^2 + b^2)$$

where a is the length and b is the thickness of the rectangular-type rod. However, if the length, a, is much greater than the thickness, b, then the following can be used as a very good approximation of the moment of inertia around the center of mass:

$$I_{cm} = \frac{1}{12}mL^2$$

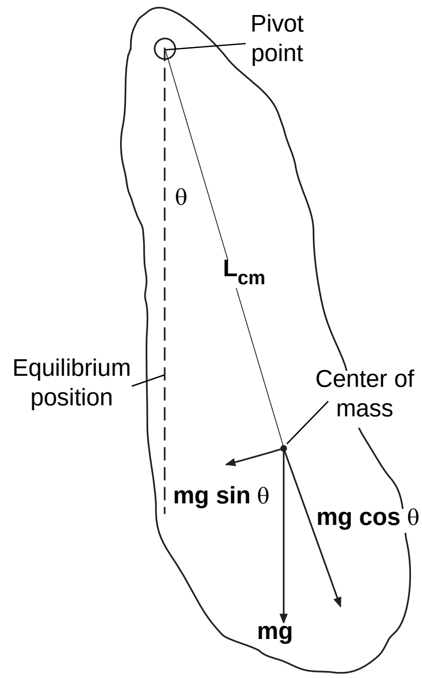


Figure 11.5: Physical Pendulum

where m is the mass and L is the length of the rod.

If the rod pivots around any other axis that is parallel to the axis through the center of mass, the Parallel Axis Theorem states that the moment of inertia about the parallel axis, $I_{parallel}$, is the sum of the moment of inertia around the center of mass, I_{cm}, plus mL_{cm}^2, where m is the mass of the rod and L_{cm} is the perpendicular distance from the center of mass to the pivot point, or

$$I_{parallel} = I_{cm} + mL_{cm}^2 = \frac{1}{12}mL^2 + mL_{cm}^2$$

The formula for the period of oscillation becomes

$$T = 2\pi\sqrt{\frac{\frac{1}{12}L^2 + L_{cm}^2}{L_{cm}g}}$$

At what distance, L_{cm}, does the period of oscillation, T, become a *minimum*?

In this experiment you will determine the distance, L_{cm}, from the pivot point to the center of mass that gives the minimum period of oscillation for the physical pendulum.

Procedure

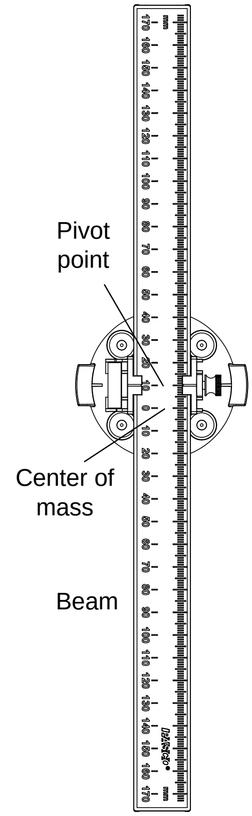


Fig. 11.6: Setup

1. Move the pivot of the Balance Arm to a position one centimeter (cm) above the midpoint of the beam (presumably the center of mass of the beam). Mount the pivot on the Statics Board.

2. Start the beam swinging but keep the angle of the swing reasonably small (less than 20°)..

3. Measure and record the total time for 10 oscillations.

4. Change the position of the beam so that the pivot point is one centimeter farther from the center of mass. Repeat the measurement for the total time of oscillation.

5. Increase the distance, L_{cm}, by one centimeter again and repeat the procedure.

6. Increase the distance, L_{cm}, by one centimeter until you reach the 170 mm mark on the beam.

7. Record the total length, L, of the beam.

- Total length, L = ____________ m

Data Table

Trial	Distance, L_{cm} (m)	Oscillations	Total Time (s)	Measured Period (s)
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				

Calculations

1. For each distance, L_{cm}, calculate and record the Measured Period by dividing the total time by the number of oscillations.

2. Create a graph of Measured Period versus distance, L_{cm}, to determine which length gives the minimum period.

- Minimum L_{cm} = ________________

3. Use calculus to find the derivative of the period:

$$T = 2\pi\sqrt{\frac{\frac{1}{12}L^2 + L_{cm}^2}{L_{cm}g}}$$

- Set the derivative equal to zero and solve for L_{cm} to confirm that this distance is $L_{cm} = \frac{1}{\sqrt{12}}L$.
- Calculate and record the value for L_{cm} based on the total length, L.
- Theoretical L_{cm} = ________________

Questions

1. Based on the graph, for which distance from the pivot point to the center of mass (L_{cm}) is the period a minimum?

2. How does the value from the graph for the distance that gives minimum period compare to the theoretical (calculated) value for the distance?

3. What would happen if you move the pivot point to the beam’s center of mass and then pulled the beam aside at a small angle less than 20°? Assume that the pivot is perfectly frictionless.

Exp. 11C: Simple Harmonic Motion–Beam on a Spring

Equipment Needed

Item	Item
Statics Board	Mounted Spring Scale
Mass and Hanger Set	Balance Arm and Protractor
Stopwatch (ME-1234)	Thread

Theory

Imagine a horizontal beam that is supported by a hinge at one end and a vertical spring at the other end. If the beam is displace, the spring exerts a restoring force, $\boldsymbol{F} = -k\boldsymbol{x}$, to return the beam to its equilibrium position. The beam will oscillate up and down with a period, T_{beam}.

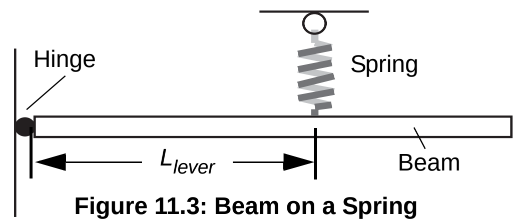


Figure 11.3: Beam on a Spring

For a *mass* on a spring, the period, T, is as follows:

$$T = \sqrt{\frac{M}{k}}$$

where M is the oscillating mass and k is the spring constant. What is the period for a *beam* on a spring?

The beam rotates at the hinge as the spring oscillates up and down. The force of the spring on the beam, $\boldsymbol{F} = -k\boldsymbol{x}$, produces a torque on the beam. Let L_{lever} be the length of the lever arm of the beam. The torque due to the spring is $\tau = \boldsymbol{F}L_{lever}$. A net torque causes angular acceleration, α, that is directly proportional to the torque, τ, and inversely proportional to the moment of inertia, I. That is,

$$\alpha = \frac{\tau}{I}$$

or $\tau = \alpha I$. Setting the two expressions for torque equal to each other gives $\boldsymbol{F}L_{lever} = \alpha I$ or $-k\boldsymbol{x}L_{lever} = \alpha I$ where $\boldsymbol{x}$ is the displacement of the spring up and down as it oscillates.

The angular acceleration, α, and the tangential (linear) acceleration, a_T, of the beam are related. The tangential acceleration, $a_T = \alpha r$ where r is the radius of rotation. In this case, the radius of rotation is the lever arm, L_{lever}, so $a_T = \alpha L_{lever}$, or $\alpha = a_T/L_{lever}$.The expression becomes:

$$-kxL_{lever} = \frac{a_T}{L_{lever}}I$$

Solving for a_T gives:

$$a_T = \frac{kL_{lever}^2}{I}x$$

This expression has the form of $a_T = \omega^2 x$, where ω is the angular frequency, so ω is:

$$\omega = \sqrt{\frac{kL_{lever}^2}{I}} = L_{lever}\sqrt{\frac{k}{I}}$$

Since the angular frequency, $\omega = 2\pi/T$, the period, $T = 2\pi/\omega$, or:

$$T = \frac{2\pi}{L_{lever}}\sqrt{\frac{I}{k}}$$

where I is the moment of inertia and k is the spring constant. If you assume that the beam is like a rectangular-type rod, then the moment of inertia about the center of mass of the rod is:

$$I_{cm} = \frac{1}{12}ML^2$$

where M is the mass and L is the length of the rod. The Parallel Axis Theorem predicts that the moment of inertia about an axis at the end of the rod is

$$I_{parallel} = I_{cm} + ML_{cm}^2 = \frac{1}{12}ML^2 + ML_{cm}^2$$

where L_{cm} is the distance from the center of mass of the rod to the parallel axis (the pivot point).

Of course, any other mass added to the beam would alter the moment of inertia. For example, if a point mass, m, is at a distance, r, from the pivot point, its moment of inertia is $I = mr^2$. The moment of inertia would be the sum.

Procedure

1. Measure and record the mass of the of the Balance Arm beam. Measure and record the mass of a Protractor.

2. Place the Pivot at the 170 mm mark on the Balance Arm beam. Mount the Protractor on the beam at the 110 mm mark. Mount the Balance Arm beam on the Statics Board.

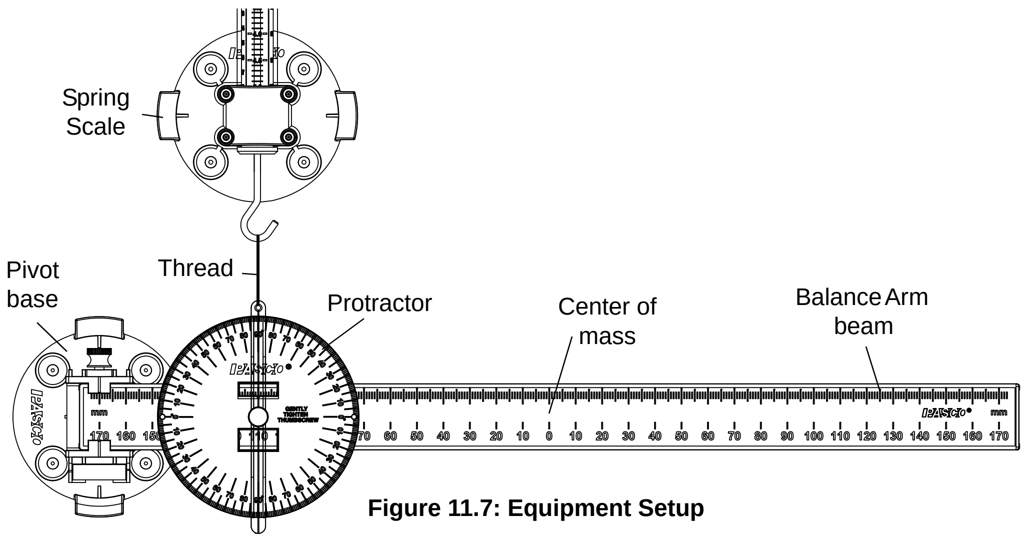


Figure 11.7: Equipment Setup

3. Mount the Spring Scale on the Statics Board so that the scale is perfectly vertical. Use thread to connect the Angle Indicator on the Protractor to the bottom hook of the Spring Scale.

4. Adjust the Balance Arm and the Spring Scale so that the arm is horizontal and the scale is directly above the center of the protractor.

5. Push the Balance Arm beam down to a small angle (less than 20°) and release it smoothly. Measure the time for ten complete oscillations (or as many oscillations as possible). Record the number of oscillations and the total time.

6. Calculate and record the Measured Period, $T_{measured}$, by dividing the total time by the number of oscillations.

7. Repeat the measurement for the first position of the Protractor five times.

8. Change the position of the Protractor by moving it 10 millimeters (mm) farther away from the pivot point and repeat the measurements.

9. Repeat the procedure for 90 mm, 80 mm, and 70 mm for a total of five distances.

10. Measure and record the length of the beam, L, and the distance from the pivot point to the center of mass (c.o.m.) of the beam, L_{cm}.

11. See the experiment Hooke's Law to find the spring constant, k, of the Spring Scale and record its value.

Calculations

1. Calculate and record the Measured Period for each trial.

2. Calculate and record the Average Measured Period for each lever arm length.

Assume that the Protractor is a point mass, m, at a distance r from the axis of rotation. Its moment of inertia added to the beam's moment of inertia gives the following:

$$I = I_{beam} + mr^2 = \frac{1}{12}ML^2 + ML_{cm}{}^2 + mr^2$$

Since the distance r from the axis of rotation is also the length of the lever arm, L_{lever}, the moment of inertia becomes:

$$I = I_{beam} + mr^2 = \frac{1}{12}ML^2 + ML_{cm}{}^2 + mL_{lever}{}^2$$

and the period of oscillation, T, becomes:

$$T = \frac{2\pi}{L_{lever}}\sqrt{\frac{I}{k}} = \frac{2\pi}{L_{lever}}\sqrt{\frac{\frac{1}{12}ML^2 + ML_{cm}{}^2 + mL_{lever}{}^2}{k}}$$

3. Calculate the Theoretical Period, $T_{theoretical}$, for each trial and record the result.

Data

- mass of beam, M = ____________ mass of protractor, m = ________________
- length, L = _____________ distance from pivot point to c.o.m., L_{cm} = ________________
- spring constant, k = ________________

Data Table

L_{lever} (m)	Oscillations	Total Time (s)	Measured Period (s)	Theoretical Period (s)
0.060				
Average Measured Period (s)				
0.070				
Average Measured Period (s)				
0.080				
Average Measured Period (s)				
0.090				
Average Measured Period (s)				
0.100				
Average Measured Period (s)				

Questions

1. How well does the theoretical value for the period of oscillation compare to the average measured period of oscillation for each trial?

2. Does the equation for the period of a beam on a spring provide a good mathematical model for the actual period of a beam on a spring? Why or why not?

Extension

- Plot a graph of period, T, versus the reciprocal of L_{lever}. Determine the slope of the line and compare the slope's value to the following:

$$2\pi\sqrt{\frac{\frac{1}{12}ML^2 + ML_{cm}{}^2 + mL_{lever}{}^2}{k}}$$

Exp. 12: Simple Machines–The Lever

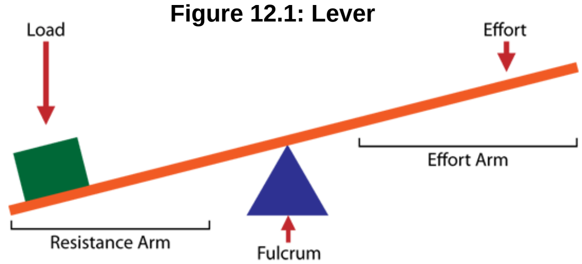
Equipment Needed

Item	Item
Statics Board	Mounted Spring Scale
Mass and Hanger Set	Balance Arm and Protractors
Large Pulley	Thread
Pencil or Dry Erase Marker Pen	

Theory

The workings of a lever can be understood using the concept of torque. When the torque produced by the applied force (called the "effort") becomes greater than the torque of the object being lifted (called the "load"), the lever will rotate about its pivot point (sometimes called the "fulcrum"), raising the load. However, levers can also be explained in terms of work and the conservation of energy.

Figure 12.1: Lever



In physics, the precise mathematical definition of work is the force applied to an object multiplied by the distance over when that force acts, or $W = \boldsymbol{F}\boldsymbol{d}$ where W is the work, $\boldsymbol{F}$ is the applied force, and $\boldsymbol{d}$ is the displacement of the object in the direction of the force. (If a force is applied to an object, but the object does not move in the direction of the force, then, technically, no work is done.) Whenever work is done on an isolated system, the energy of the system will change by exactly the amount of work that was performed, or $\Delta E = W$ where ΔE is the change of energy.

In this experiment, you will apply a measurable amount of work to a lever and observe the change in the gravitational potential energy of the load.

Setup

1. Put a Protractor at each end of the Balance Arm beam and mount the Balance Arm on the Statics Board with the beam centered in the pivot.

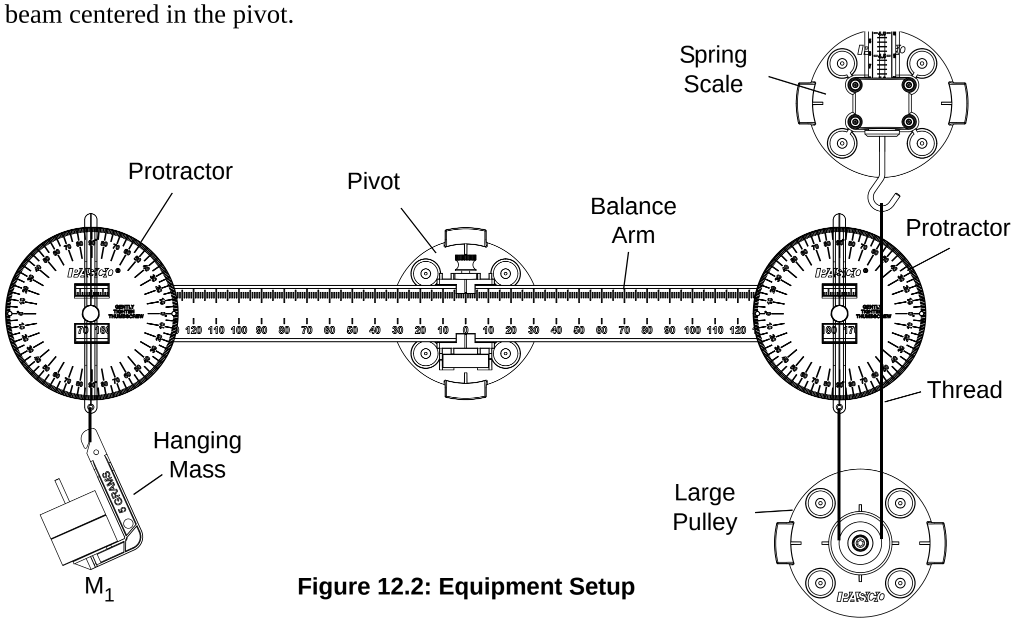


Figure 12.2: Equipment Setup

2. Mount a Large Pulley and Spring Scale at one end of the Balance Arm and use thread to connect the Spring Scale to the Protractor on the Balance Arm.

3. Use thread to hang a mass hanger from the other Protractor and add 200 g of mass to the hanger.

4. Adjust the Large Pulley and the Spring Scale so that the Balance Arm beam is horizontal and level.

Procedure

1. Make and record the necessary measurements to show that the torques produced by the Spring Scale and the weight of the Hanging Mass are balanced.

2. Use a pencil or dry erase marker pen to outline the base of the Spring Scale and the position of the top of the Hanging Mass on the Statics Board.

3. Slowly push the Spring Scale upward. (If you perform this movement slowly enough, the reading on the Spring Scale will not vary appreciably.)

4. Mark the new positions of the Hanging Mass and the Spring Scale.

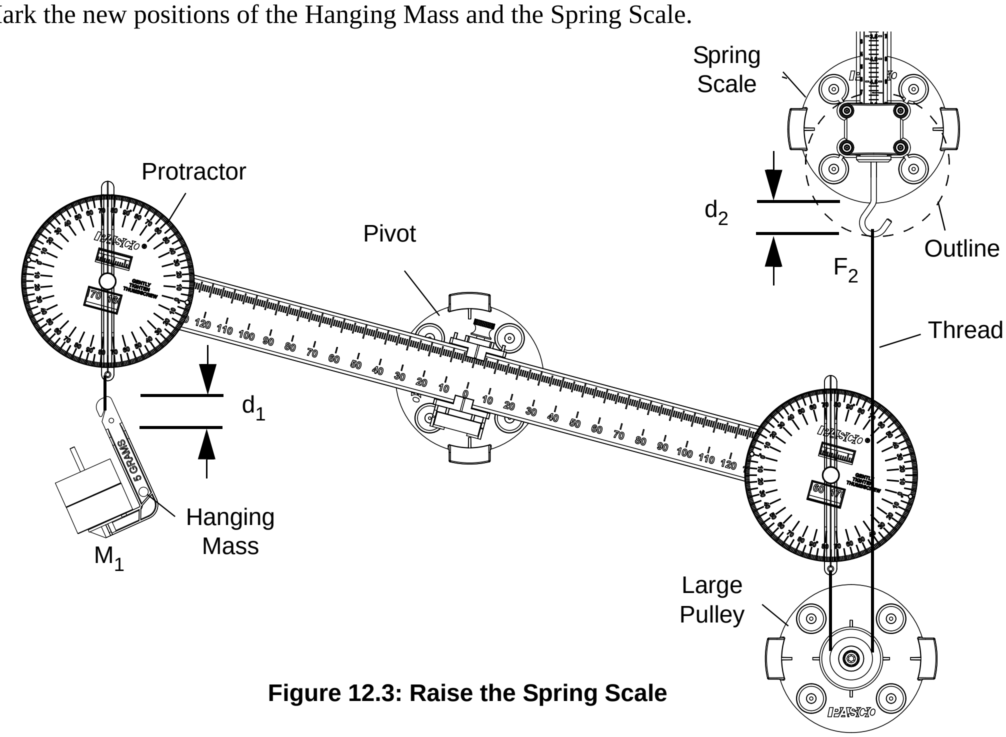


Figure 12.3: Raise the Spring Scale

5. Measure and record the distances, d_1 (distance Hanging Mass moved) and d_2 (distance Spring Scale moved). Also record F_2, the reading on the Spring Scale, and M_1 and W_1, the mass and weight of the Hanging Mass.

6. Move the protractor with the hanging mass to a new location about halfway to the pivot point and repeat the experiment. Measure and record the new values for d_1, d_2, and F_2.

Data Table

Item	Trial 1	Trial 2	Item	Trial 1	Trial 2
Distance hanging mass moved, d_1			Mass of hanging mass, M_1		
Distance Spring Scale moved, d_2			Weight of hanging mass, W_1		
Force of Spring Scale, F_2					

Calculations

1. Calculate and record the work done on the system as you raised the Spring Scale, where Work = F_2 d_2.

2. Calculate and record the change in potential energy of the Hanging Mass as it was raised in the Earth's gravitational field, where $\Delta E_{potential} = M_1 \, g \, d_1$ and g = 9.8 N/kg.

Trial	Work	Change of potential energy
1		
2		

Questions

1. How did the work done on the system compare to the change in potential energy of the system?

2. How does a lever make it easier to perform work, such as raising a heavy load? Explain in terms of conservation of energy.

Extension

The type of lever that has the fulcrum between the load (hanging mass) and the applied force (Spring Scale) is called a Class I Lever. A Class II Lever has the load between the fulcrum and the applied force, and the Class III lever has the applied force between the fulcrum and the load.

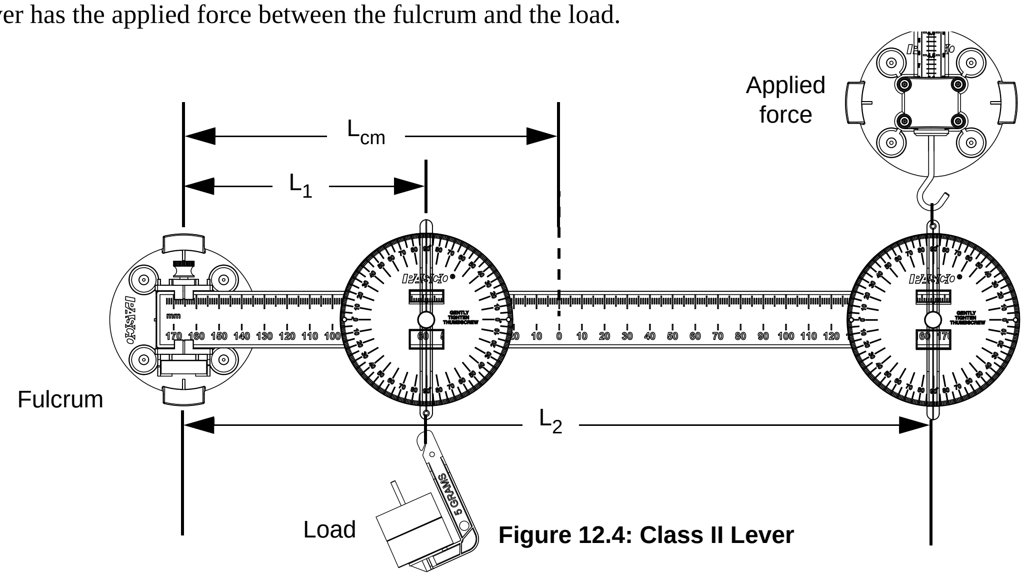


Figure 12.4: Class II Lever

A wheelbarrow is an example of a Class II Lever, and the human forearm is an example of a Class III Lever.

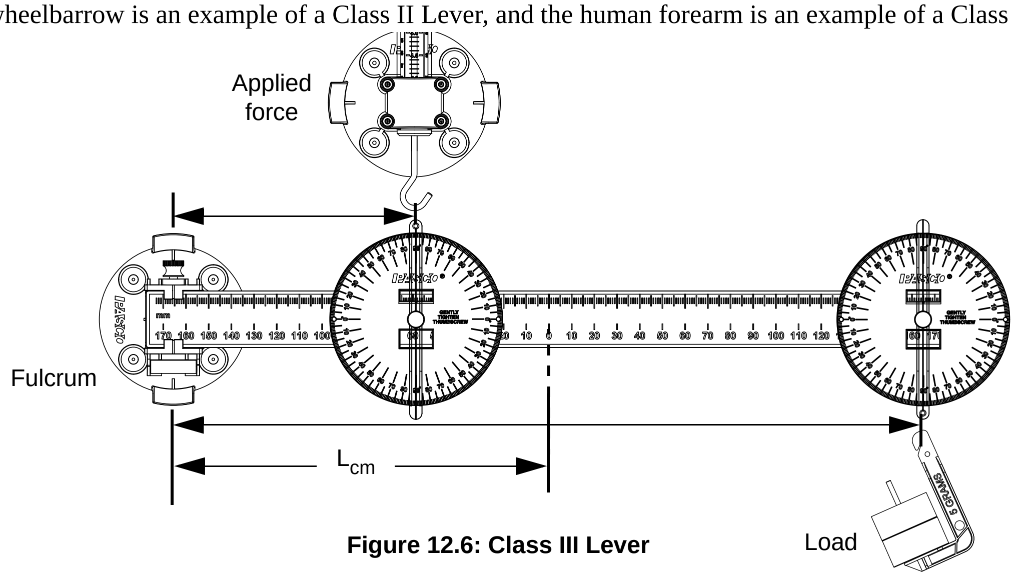


Figure 12.6: Class III Lever

When a lever is in equilibrium, the sum of the clockwise torques about the pivot point (fulcrum) is equal to the sum of the counterclockwise torques about the pivot point.

Make and record the measurements that are necessary to determine if the sum of the clockwise torques equals the sum of the counterclockwise torques for the Class II and Class III levers. Will you need to take the mass of the protractors and the mass of the Balance Arm beam into account?

Diagram your results and show your measurements and calculations on a separate sheet of paper.

Data Table

Table 16.1:

Lever	Σ clockwise torque	Σ counterclockwise torque	Net torque
Class II			
Class III			

Questions

1. For the Class II Lever, is the net torque equal to zero when the lever is in equilibrium?

2. For the Class III Lever, is the net torque equal to zero when the lever is in equilibrium?

Exp. 13: Simple Machines–The Inclined Plane

Equipment Needed

Item	Item
Statics Board and Pulley	Mounted Spring Scale
Mass and Hanger Set	Inclined Plane and Mass Cart
Pencil or Dry Erase Marker Pen	Thread

Introduction

The inclined plane, like the lever, is often used to help raise heavy objects. In a previous experiment, you analyzed this use of the inclined plane in terms of the forces that are involved. In this experiment you will take a second look at the inclined plane, using the concepts of work and conservation of energy as you applied them in previous experiments.

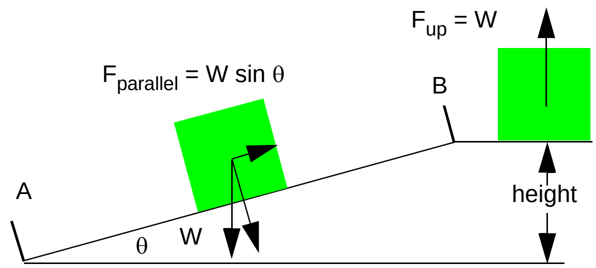


Figure 13.1: Inclined Plane

The work to lift an object with a weight, *W*, is the product of the weight and the height. To push the same object up the inclined plane requires a smaller force, *W sin θ*, than the object's weight, but the distance over which the force acts is longer. The angle *θ* is the incline of the plane.

Procedure

1. Put a 100 g mass on the peg of the Mass Cart and use the Spring Scale to measure the total weight, *W*, of the cart plus mass.

- Weight, *W* = _______________

2. Put the Inclined Plane on the Statics Board at a relatively small angle (such as 15°). Put a Pulley and the Spring Scale on the board near one end of the Inclined Plane. Put the Mass Cart on the Inclined Plane and use thread to connect the end of the Mass Cart to the Spring Scale.

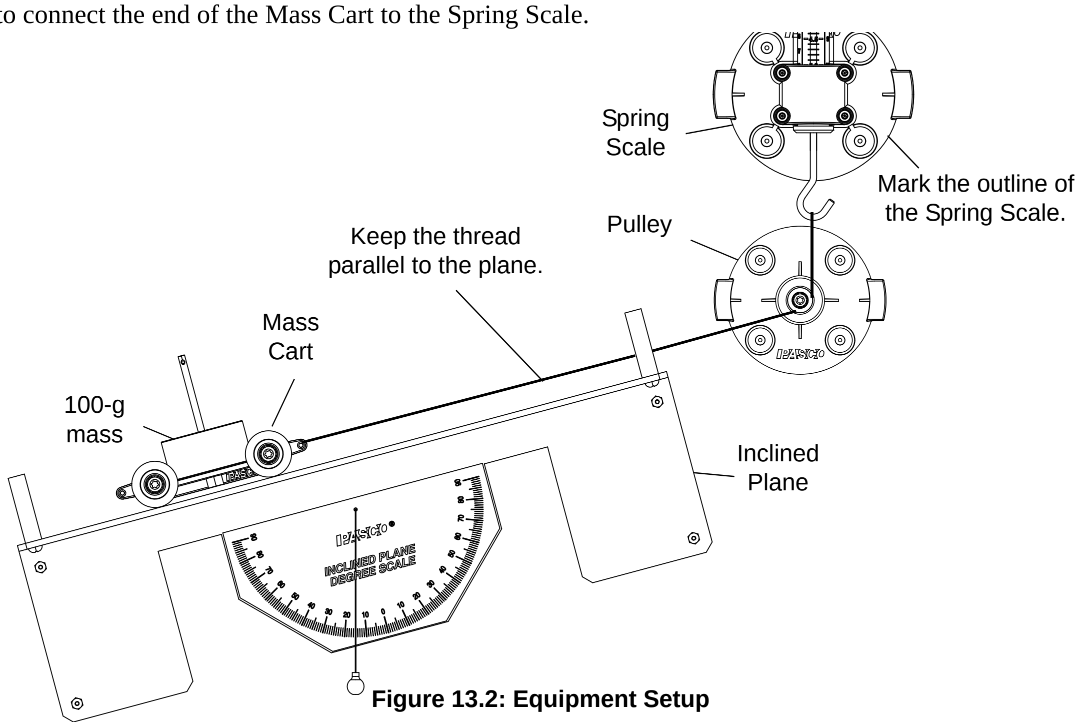


Figure 13.2: Equipment Setup

3. Measure and record the magnitude of the force, F_1, exerted by the Spring Scale on the Mass Cart, and the angle θ of the Inclined Plane.

- Force, F_1 = _____________ Angle, θ = _______________

4. Use a pencil or dry erase marker pen to outline the base of the Spring Scale on the Statics Board.

5. Slowly raise the Spring Scale – slowly enough that there is no appreciable change in the reading on the Spring Scale.

6. Measure and record the distance, d_1, that the Spring Scale pulled the Mass Cart.

- Distance, d_1 = _______________

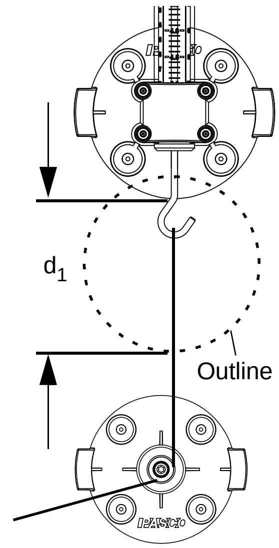


Fig. 13.3: Measure d_1

Calculations

1. Calculate and record the height that the Mass Cart was lifted as it was pulled up the Inclined Plane by the Spring Scale.

- Height, $d_1 \sin\theta$ = ________________

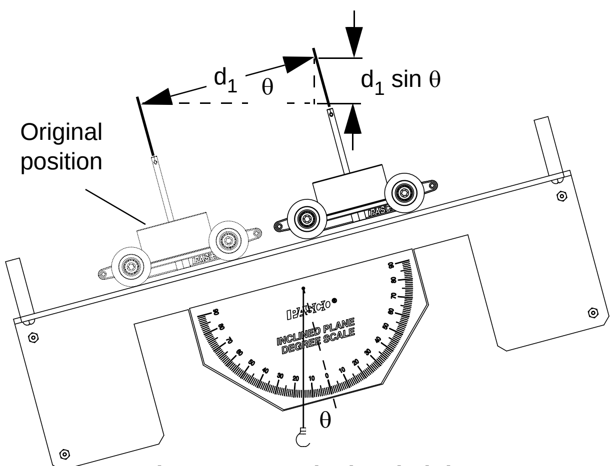


Figure 13.4: Calculate height

2. Calculate and record the amount of work done by the Spring Scale.

- Work = $F_1 d_1$ = _________________.

3. Calculate and record the change in gravitational potential energy of the Mass Cart.

- ΔE = Weight x height = $W d_1 \sin\theta$ = __________________.

Question

- How does the change in gravitational potential energy of the Mass Cart compare to the work done by the Spring Scale?

Procedure

1. Carefully hang the Mass Cart plus the 100 g mass from the Spring Scale. Record the force, F_2, exerted by the Spring Scale.

- Force, F_2 = ______________

2. Slowly push the Spring Scale straight up a distance, $d_1 \sin \theta$, the height to which the Mass Cart was raised when it was on the Inclined Plane. (Raise the Spring Scale slowly so that there is no appreciable change in the reading of the Spring Scale.)

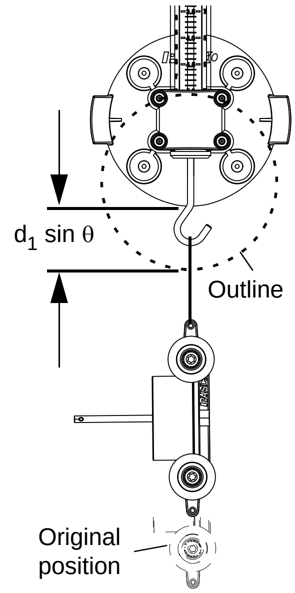


Fig. 13.5: Raise the Mass Cart

Calculations

1. Calculate and record the amount of work that was done on the Mass Cart by lifting it straight up by a distance of $d_1 \sin \theta$.

- Work = F d = $F_2\, d_1 \sin \theta$ = ____________________

2. Calculate and record the change in gravitational potential energy of the Mass Cart.

- ΔE = Weight x height = $W\, d_1 \sin \theta$ = ___________________.

Questions

1. How does the work done on the Mass Cart when it was on the Inclined Plane compare to the work done when it was lifted directly by the Spring Scale?

2. How does an inclined plane make it easier to perform work, such as raising a heavy load? In other words, what is the advantage of using the inclined plane?

Exp. 14: Simple Machines–The Pulley

Equipment Needed

Item	Item
Statics Board and Pulley	Mounted Spring Scale
Large Pulley and Small Pulleys (2)	Double Pulley Block
Mass and Hanger Set	Thread

Theory

In previous experiments, you used pulleys to change the direction of applied forces. However, systems of pulleys can be arranged to translate relatively small applied forces into much larger forces, much the same way as a lever or inclined plane. In this experiment you will take a second look at pulleys and investigate how systems of pulleys can be used to amplify the applied force as work is done.

In an ideal pulley system there would be not friction in the pulleys. For example, the applied force would be transferred completely to the hanging mass.

As with the lever and the inclined plane, pulley systems can be understood by analyzing either the forces acting on the system or the work performed on and by the system. In this experiment you will investigate several pulley systems.

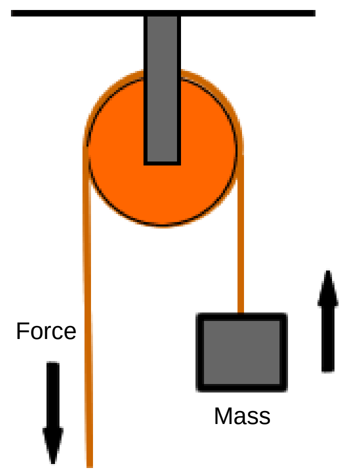


Fig. 14.1: Simple Pulley

Procedure

The effects of friction are more noticeable in this experiment than with the lever or the inclined plane. Start by investigating the effects of friction in the pulleys.

1. Put 200 g of mass on a mass hanger and use the Spring Scale to measure the total weight, *W*, of the mass hanger plus mass. Record your measurement.

- Weight, *W* = _______________

2. Put the Spring Scale and two Pulleys on the Statics Board as shown and use thread to attach the Spring Scale to the hanging mass. Record the reading of the force, *F*, on the Spring Scale.

- Force, *F* = ________________

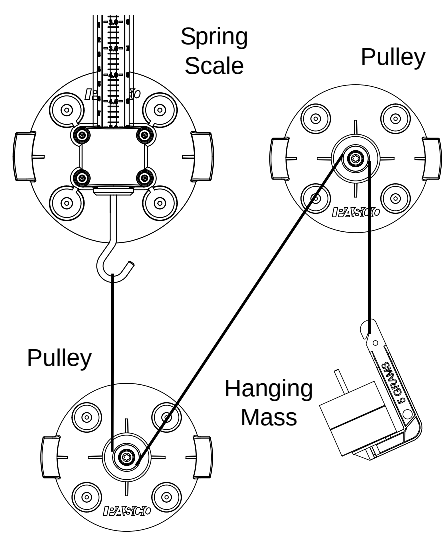


Figure 14.2: Equipment Setup

Question

- How does the force reading, *F*, on the Spring Scale for the pulley setup compare to the weight, *W*, of the hanging mass?

3. Set up each of the three pulley systems shown below. For each pulley system, perform work on the system by slowing raising the Spring Scale. Measure and record the following:

Symbol	Description	Symbol	Description
F	Force reading on the Spring Scale	d_2	Distance that the hanging mass is raised
W/F	Ratio of weight of hanging mass to force	Work	Work done by raising the Spring Scale ($F \times d_1$)
d_1	Distance that the Spring Scale is raised	ΔE	Change in gravitational potential energy ($W \times d_2$)

4. Use a pencil or dry erase marker pen to outline the base of the Spring Scale. Also mark the position of the top of the hanging mass.

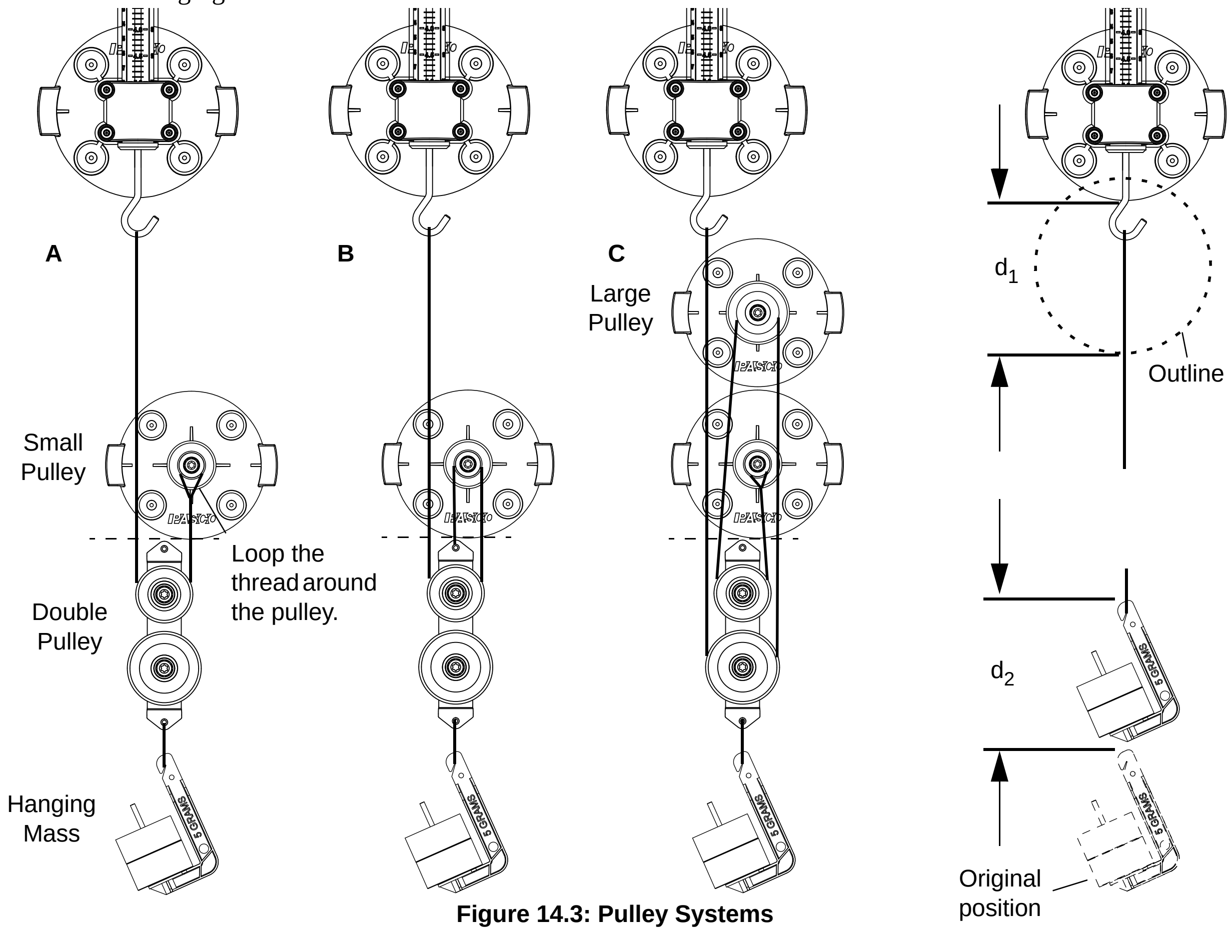


Figure 14.3: Pulley Systems

Note that the drawings are not to scale. Allow more vertical distance between the components.

5. Slowly raise the Spring Scale – slowly enough that there is no appreciable change in the reading on the Spring Scale.

6. Measure and record the distance, d_1, that the Spring Scale was raised. Also measure and record the distance, d_2, that the hanging mass was raised.

Data Table

System	W	F	W/F	d_1	d_2	Work	ΔE
A							
B							
C							

Questions

1. How does the relationship of weight (W) and force (F) compare to the relationship of the distance the Spring Scale was raised (d_1) and the distance the hanging mass was raised (d_2)?

2. Compare the value of W/F to the number of threads that crosses the dotted line in the figure of the pulley system.

Exp. 15: Forces on a Boom

Equipment Needed

Item	Item
Statics Board	Mounted Spring Scale and Pulley
Balance Arm and Protractors)	Thread
Mass and Hanger Set	

Theory

A boom supported by a cable has a mass suspended at its upper end. The lower end of the boom is supported by a pivot.

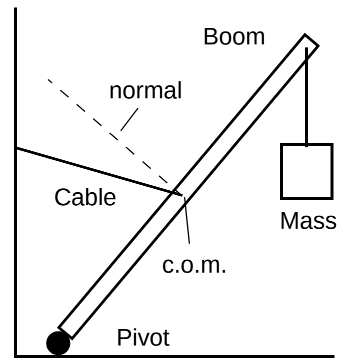


Fig. 15.1: Suspended Mass

For example, if the mass is 100 kg and the boom has a mass of 50 kg, what is the tension, T, in the cable? Assume that the cable is attached at the boom's center of mass and is at an angle relative to the boom. The boom is at an angle of 50° to the horizontal.

For the boom to be in equilibrium, all of the translational forces (F_x and F_y) and all of the torques must add up to zero. One torque is produced by the tension in the cable. Another torque is produced by the weight of the beam. A third torque is produced by the weight of the hanging mass.

$$\Sigma\tau_{clockwise} = (WL\cos\beta) + (W_{cm}L_{cm}\cos\beta)$$
$$\Sigma\tau_{counterclockwise} = TL_{cm}\cos\alpha$$

where W is the weight of the hanging mass, L is the lever arm from the pivot point to the place where the hanging mass is attached, W_{boom} is the weight of the boom, L_{cm} is the lever arm from the pivot point to the center of mass, β is the angle of the boom, T is the tension in the cable, and α is the angle of the cable relative to the normal of the boom.

If the boom is in equilibrium, the net torque is zero. The expression for the tension, T, in the cable becomes:

$$T = \frac{(WL\cos\beta) + (W_{cm}L_{cm}\cos\beta)}{L_{cm}\cos\alpha}$$

Procedure

1. Set up the Balance Arm with the pivot at one end, a protractor at the midpoint, and a second protractor at the other end. Mount the Balance Arm in a lower corner of the Statics Board.

2. Mount the Spring Scale and a Pulley on the board and use thread to attach the Spring Scale to the protractor at the midpoint of the beam.

3. Use thread to attach a hanging mass to the protractor at the end of the beam.

4. Make and record the necessary measurements.

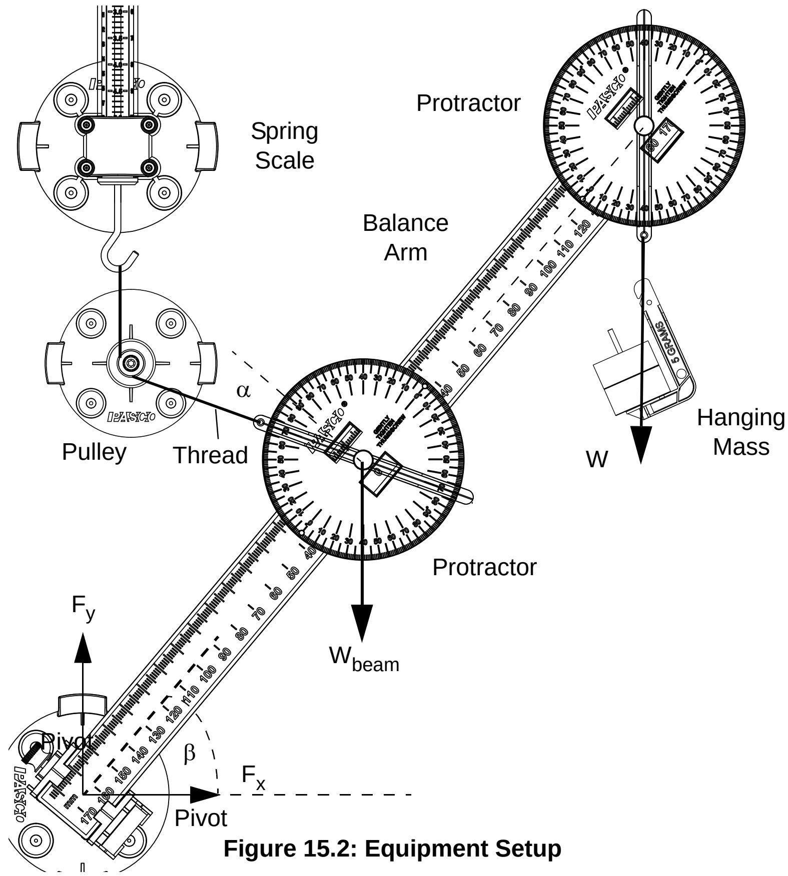


Figure 15.2: Equipment Setup

Data Table

Item	Value
Angle, β, of Balance Arm	
Angle, α, of thread	
Weight, W, of hanging mass	
Weight, W_{beam}, of beam	
Weight of protractor	
Lever arm, L_{cm}, pivot to c.o.m.	
Lever arm, L, to hanging mass	
Tension, T, in the thread	

Calculations

1. Calculate the sum of the clockwise torques about the pivot point.

- Σ torque$_{clockwise}$ = ______________________

2. Calculate the theoretical tension, T, in the thread.

- Tension$_{theoretical}$ = _________________________

Question

- How does the force reading on the Spring Scale compare to the theoretical value for the tension in the cable?

Exp. 16: Modified Atwood's Machine

Equipment Needed

Item	Item
Statics Board	Small Pulley (2)
Mass and Hanger Set	Thread
Stopwatch	Dry-erase Marker Pen or Pencil

Theory

The acceleration of an object depends on the net applied force and the object's mass. In an Atwood's Machine, the difference between two hanging masses determines the net force acting on the system of the two masses. The net force accelerates both of the hanging masses; the heaver mass is accelerated downward and the lighter mass is accelerated upward.

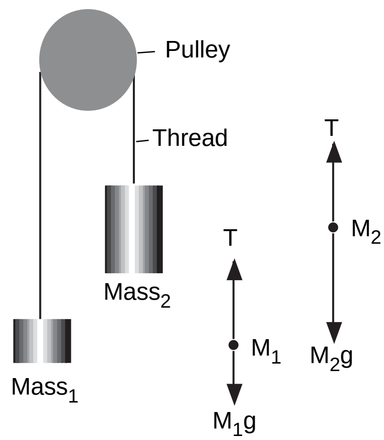


Fig. 16.1: Atwood's FBD

The Atwood's Machine was invented in 1784 by the Reverend George Atwood to demonstrate the principles of acceleration and net force. In an ideal Atwood's Machine, two unequal masses are attached to a flexible, massless string which passes over a frictionless, massless pulley. A real Atwood's Machine is not as simple as its ideal counterpart. Strings and pulleys are not massless and pulleys are not frictionless. The rotational inertia of the pulley complicates the demonstration.

In the free body diagram (FBD) of the Atwood's Machine, T is the tension in the string, $Mass_2$ is greater than $Mass_1$, and g is the acceleration due to gravity. Using the convention that up is positive and down is negative, the net force equations for the two masses are:

$$T_1 - M_1 g = F_{net} = M_1 a$$
$$T_2 - M_2 g = F_{net} = M_2(-a)$$

Ideally (where the string is massless and doesn't stretch and the pulley is massless and frictionless), the tension, *T*, is the same for both hanging masses. Let $T_1 = T_2$ and solve for the theoretical acceleration, *a*:

$$a = g\frac{M_2 - M_1}{M_2 + M_1}$$

The theoretical acceleration is the difference in the two forces (M_2g - M_1g) divided by the sum of the two masses.

In this experiment you will investigate the acceleration of the masses in an Atwood's Machine. You can determine the acceleration, *a*, by measuring the time, *t*, it takes for one of the masses to fall a known distance, *d*.

$$a = \frac{2d}{t^2}$$

Compare your measured acceleration to the theoretical acceleration.

Setup

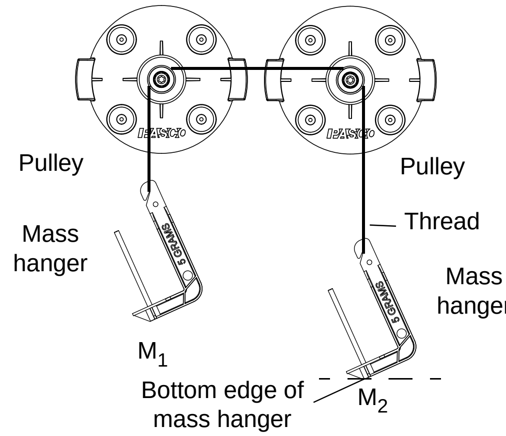


Figure 16.2: Equipment Setup

1. Place the two Small Pulleys on the Statics Board near the top edge of the board and close to each other. Make sure that the two pulleys are level.

2. Connect a thread from one Mass Hanger to a second Mass Hanger so the hangers are suspended over the two pulleys.

- NOTE: Make the thread long enough so that the heavy mass almost reaches the base of the Statics Board when the light mass is almost up to the pulley.

3. Pull the M_1 mass hanger down so that the top of the M_2 mass hanger is just below the base of the pulley. Use a dry-erase marker or a pencil to put a horizontal mark on the Statics Board for the position of the bottom edge of the M_2 mass hanger.

4. Measure vertically down from the first mark a distance of 20 cm (0.020 m) and put a second horizontal mark on the board.

Procedure

Part 1: Keep Total Mass Constant

1. Put the following on the 'light' (M_1) mass hanger: 1 x 50-g, 3 x 20-g, 2 x 5-g, 2 x 2-g, 1 x 1-g. Put the following on the 'heavy' (M_2) mass hanger: 1 x 50-g, 3 x 20-g, 1 x 10-g, 1 x 5-g.

2. Including each 5-g mass hanger, record the total mass for M_1 and the total mass for M_2 in the Data Table.

3. Move the 1-g mass from the light mass hanger to the heavy mass hanger (so the difference in mass is 1 g). Pull the light mass hanger down so that the bottom edge of the heavy mass hanger is aligned with the starting mark on the Statics Board.

- NOTE: Stop the hangers from spinning or swinging.

4. Release the light mass hanger and start timing at the same instant. Stop timing when the bottom edge of the heavy mass hanger reaches the finish mark on the Statics Board. Record the time, t_1, in the Data Table. Repeat the trial two more times and record the results.

5. For the second trial, return the 1-g mass to the light hanger and move one of the 2-g masses to the heavy hanger (ΔM = 2 g). Repeat the data recording procedure three times.

6. For the third trial, move the 1-g mass from the light hanger to the heavy hanger (ΔM = 3 g) and repeat the procedure three times.

7. For the fourth trial, move the 1-g mass from the heavy hanger back to the light hanger and move the 2-g mass from the light hanger over to the heavy hanger (ΔM = 4 g) and repeat the procedure three times.

8. For the fifth trial, move the 1-g mass from the light hanger to the heavy hanger (ΔM = 5 g) and repeat the procedure three times.

Data Table 1: Constant Total Mass

Trial	M_1 (kg)	M_2 (kg)	ΔM (kg)	t_1	t_2	t_3	t_{avg}	$a_{theoretical}$	$a_{measured}$	% diff.
1			0.001							
2			0.002							
3			0.003							
4			0.004							
5			0.005							

Part 2: Keep Net Force Constant

1. Put a 50-g mass on the 'light' (M_1) mass hanger and put a 50-g mass and a 5-g mass on the 'heavy' (M_2) mass hanger.
2. Including each 5-g mass hanger, record the total mass for M_1 and the total mass for M_2 in the Data Table.
3. Pull the light mass hanger down so that the bottom edge of the heavy mass hanger is aligned with the starting mark on the Statics Board.

- NOTE: Stop the hangers from spinning or swinging.

4. Release the light mass hanger and start timing at the same instant. Stop timing when the bottom edge of the heavy mass hanger reaches the finish mark on the Statics Board. Record the time, t_1, in the Data Table. Repeat the trial two more times and record the results.
5. For the second trial, add 20 g of mass to each hanger (but ΔM still is 5 g). Repeat the data recording procedure three times.
6. For the third trial, add another 20 g of mass to each hanger and repeat the procedure three times.
7. For the fourth trial, add another 20 g of mass to each hanger and repeat the procedure three times.
8. For the fifth trial, add another 20 g of mass to each hanger and repeat the procedure three times.

Data Table 2: Constant Net Force

Trial	M_1 (kg)	M_2 (kg)	ΔM (kg)	t_1	t_2	t_3	t_{avg}	$a_{theoretical}$	$a_{measured}$	% diff.
1			0.005							
2			0.005							
3			0.005							
4			0.005							
5			0.005							

Calculations

1. Calculate and record the average time for each trial.
2. Calculate and record the theoretical acceleration and the measured acceleration for each trial.

3. Calculate and record the percent difference of the theoretical acceleration and the measured acceleration for each trial.

$$\%\text{ diff}=\left|\frac{a_{theoretical}-a_{measured}}{a_{theoretical}}\right|\times 100$$

4. For the Constant Total Mass data, calculate the net force ($F_{net} = \Delta M$ x g) for each trial. Plot a graph of net force (vertical axis) versus measured acceleration (horizontal axis).

Questions

1. Describe the graph of net force versus measured acceleration. What does the slope of this graph represent?

2. How does the plot of net force versus measured acceleration relate to Newton's Second Law?

3. Compare the percent difference for the theoretical and measured accelerations. What are some reasons that might account for the percent difference?

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