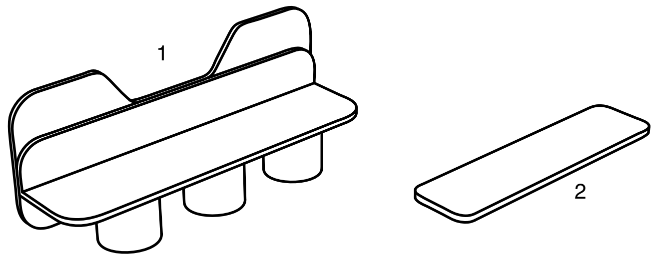
# Dynamics Cart Magnetic Damping

**ME-6828**




| **Included Equipment** | **Part Number** |
|---|---|
| 1. Magnetic Damping Accessory | ME-6828 |
| 2. Keeper plate | 648-10173 |
| **Required Equipment** | **Part Number** |
| PAScar, Plunger Cart, or Collision Cart | ME-6950, ME-9430, *or* ME-9454 |
| Aluminum Dynamics Track | ME-6953 *or similar* |

*For other recommended equipment, see experiments starting on page 3.*

## Introduction

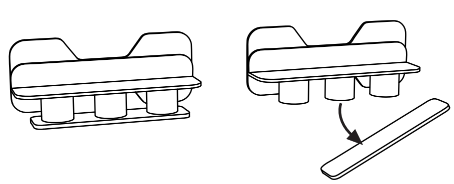
Use the Magnetic Damping Accessory to add drag to a PASCO dynamics cart. When the cart moves on an aluminum track, the accessory's magnets generate eddy currents in the track, which cause an opposing magnetic field. The resulting drag force is proportional to the cart's velocity. The amount of damping can be varied by changing the size of the gap between the magnets and the track.

This manual includes set-up instructions, three experiments (starting on page 3), and sample data (page 13).

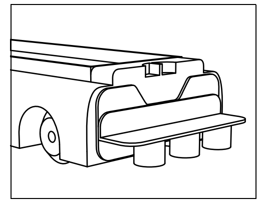
## Set-up

The Magnetic Damping Accessory attaches to a cart magnetically using the magnets built into the cart's bumper. If your cart does not have bumper magnets, install them before using the Magnetic Damping Accessory. (See instructions included with your cart for bumper magnet installation.)

**1.** Remove the keeper plate from the Magnetic Damping Accessory and put it aside.

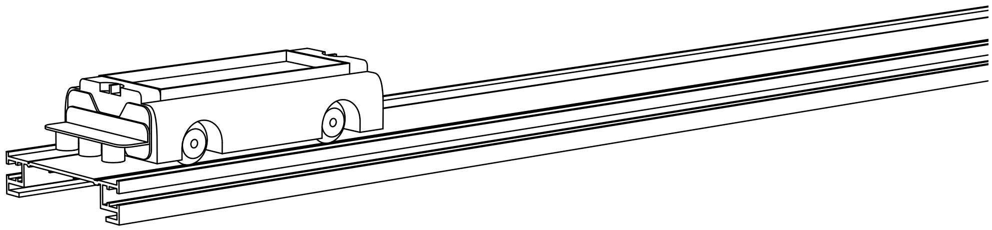


2. Place the Magnetic Damping Accessory on the magnet-equipped end of the cart as illustrated.

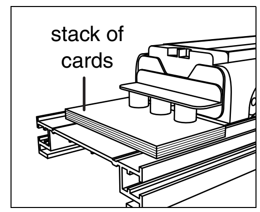


3. Place the cart on an aluminum track.

4. Slide the Magnetic Damping Accessory up or down to adjust the amount of damping. The magnets should not touch the track.



To set a gap size that can be measured or reproduced, use a stack of cards as a gauge. Place the cards between the magnets and track as illustrated. Slide the Magnetic Damping Accessory down until the magnets contact the top card; then remove the cards. Use calipers to measure the thickness of the stack. For maximum damping, use a single card.




# Storage and Safety

- The Magnetic Damping Accessory is shipped with a keeper plate attached to the magnets. The keeper protects the magnets and reduces the external magnetic field. **Always store the Magnetic Damping Accessory with the keeper in place.**

- **Use caution when replacing the keeper** to avoid being pinched or chipping the magnets.

- **Do not remove the magnets from the body of the Magnetic Damping Accessory.** If the magnets are accidentally removed, replace them with caution. The poles of all three magnets should have the same orientation.

- **Keep the Magnetic Damping Accessory away from computers and magnetic recording media.**

# Qualitative Demonstrations

The effects of magnetic damping can be surprising. Use these demonstrations illustrate the phenomenon.

- Place a strongly damped cart on a track and ask students to move it by hand. When the cart is moved quickly, the damping force is easily detected.

- Raise one end of the track by a few centimeters. Set the Magnetic Damping Accessory for maximum damping and allow the cart to run down the track. With the track inclined just enough to overcome rolling friction, the cart will creep very slowly at a constant velocity.

# Experiment 1: Damped Oscillation

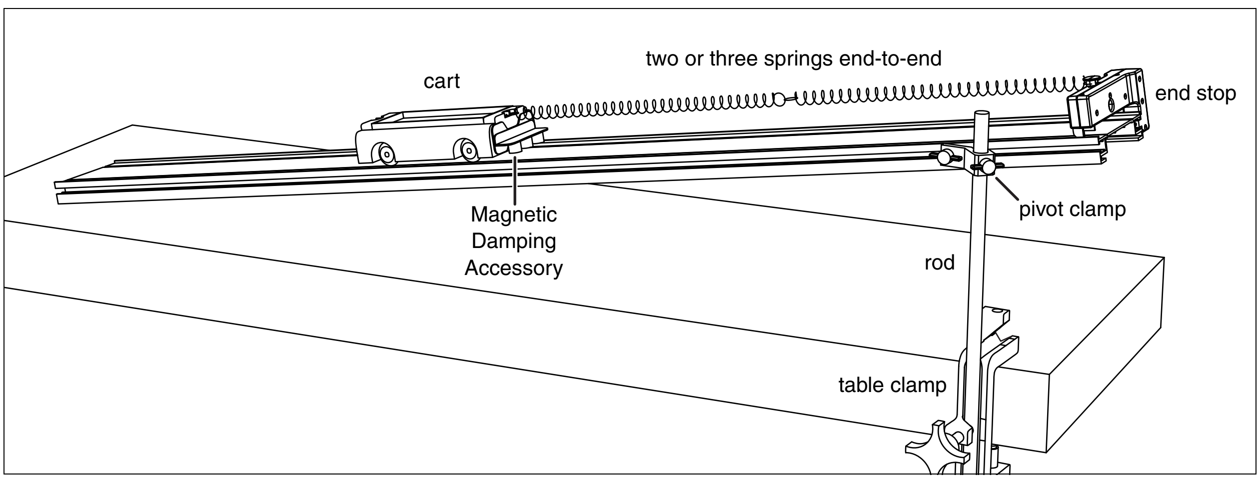
| Equipment | Part Number |
|---|---|
| Dynamics Cart Magnetic Damping | ME-6828 |
| Motion Sensor (optional for Method II) | PS-2103A *or similar* |
| Cart[1] | ME-6950, ME-9430, *or* ME-9454 |
| Aluminum Track[1] | ME-6953 *or similar* |
| Harmonic Springs[1, 2] | ME-9803A *or similar* |
| Adjustable End Stop[1] | ME-8971 *or similar* |
| IDS Pivot Clamp[1] | ME-9810 |
| Large Table Clamp | ME-9472 |
| 90 cm Rod | ME-8738 *or similar* |

*[1]These items are components of many PASCO dynamics systems.*

*[2]This experiment works best with a spring, or combination of springs, with a constant between 1.0 N/m and 1.5 N/m.*

## Set-up

1. Set up the equipment as illustrated. Use two or three weak springs combined end-to-end.




**Experiment set-up for Damped Oscillation**

2. Adjust the angle of the track (or the position of the end stop) so that the cart's equilibrium position is about 40 cm from the lower end of the track.

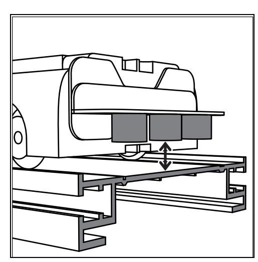
## Method I (without sensors)

*Skip to Method II if you will be using a motion sensor.*

1. Place the Magnetic Damping Accessory on the end of the cart with a gap of 10 mm between the magnets and the track.

2. Tap the cart a few times so that it moves to its equilibrium position.

3. Pull the cart up the track and hold it 20.0 cm from equilibrium.



**Gap between magnets and track**

4. Release the cart.

5. Allow the cart to oscillate. Count (and write down) the number of down-up cycles that the cart completes before stopping.

6. Repeat steps 3 through 5 with the gap set to 8 mm, 6 mm, 4 mm, and 2 mm, and with the magnets as close as possible to the track without touching.

*Skip to the Questions section on page 4.*

## Method II (using a motion sensor)

### Sensor, Interface, and Software Set-up

*For detailed information about setting up your motion sensor, interface, and software, refer to the instructions supplied with those products.*

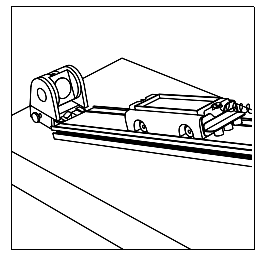
1. Place a motion sensor at the lower end of the track.

2. Adjust the angle of the track so that the cart's equilibrium position is about 40 cm from the sensor.

3. Connect the sensor to your interface.

4. Prepare a graph to display Position versus time.

5. Take some test data and adjust the sensor so that it can measure the cart's position up to a distance of about 70 cm. Delete the test data.



**Motion sensor at end of track**

*Leave the sample rate at the motion sensor's default of 10 Hz.*

### Data Collection

1. Place the Magnetic Damping Accessory on the end of the cart with a gap of 10 mm between the magnets and the track.

2. Tap the cart a few times so that it moves to its equilibrium position.

3. Pull the cart up the track and hold it 20.0 cm from equilibrium.

4. Release the cart and start data recording at the same time.

5. Allow the cart to oscillate until it comes to a stop.

6. Stop data recording.

7. Repeat steps 3 through 6 to record data runs with the gap set to 8 mm, 6 mm, 4 mm, and 2 mm. Record a final data run with the magnets as close as possible to the track without touching.

## Question

What is the effect of moving the magnets closer to the track?

# Experiment 2: Critical Damping

| Equipment | Part Number |
|---|---|
| Dynamics Cart Magnetic Damping | ME-6828 |
| Stopwatch (for Method I)<br>*or*<br>Motion Sensor (for Method II) | ME-1234 *or similar*<br><br>PS-2103A *or similar* |
| Cart[1, 2] | ME-6950, ME-9430, *or* ME-9454 |
| Aluminum Track[1] | ME-6953 *or similar* |
| Harmonic Springs[1, 3] | ME-9803A *or similar* |
| Adjustable End Stop[1] | ME-8971 *or similar* |
| IDS Pivot Clamp[1] | ME-9810 |
| Large Table Clamp | ME-9472 |
| 90 cm Rod | ME-8738 *or similar* |

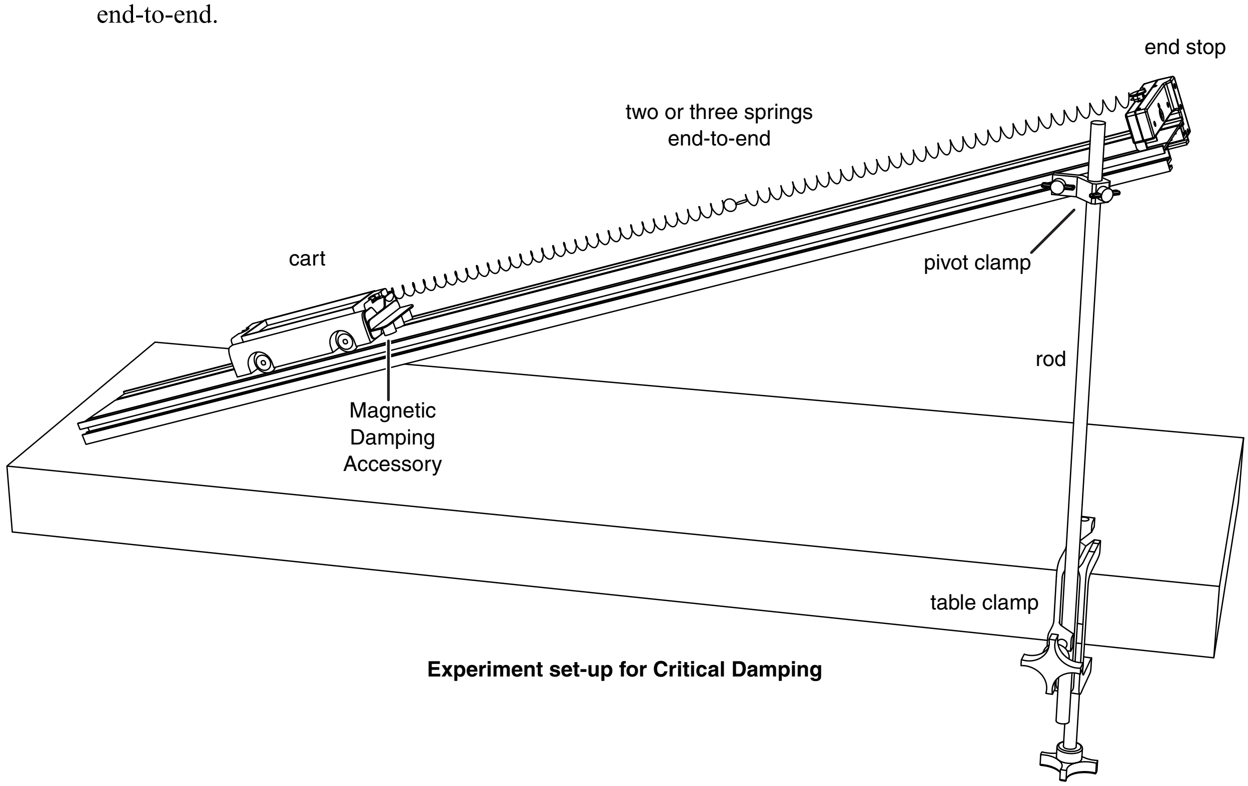
*[1]These items are components of many PASCO dynamics systems.*

*[2]Either a plastic or aluminum cart will work; however, the lighter plastic cart allows a relatively greater amount of damping.*

*[3]This experiment requires a spring or combination of springs with constant between 1.0 N/m and 1.5 N/m. If the spring constant is too high, over damping will not be achievable.*

## Set-up

1. Set up the equipment as illustrated. Use two or three weak springs combined end-to-end.




**Experiment set-up for Critical Damping**

2. Adjust the angle of the track (or the position of the end stop) so that the cart's equilibrium position is about 20 cm from the lower end of the track.

**3.** Use tape or a pencil to mark the track about 50 cm above equilibrium.

**4.** Pull the cart up the track to the mark.

**5.** Let the cart go and carefully observe it's movement as it comes to a stop. If the cart overshoots equilibrium and comes back up the track, it is under-damped. If it comes to a stop without overshooting, it is either over-damped or critically damped.

**6.** Adjust the height of the magnets to reduce or increase the amount of damping.

**7.** Repeat steps 4 through 6 several times to find the least amount of damping (biggest gap) that will make the cart stop without overshooting. This is critical damping.

## Method I (using a stopwatch)

*Skip to Method II if you will be using a motion sensor.*

**1.** Pull the cart up the track *exactly* to the mark.

**2.** Release the cart and start the stopwatch at the same time.

**3.** Stop the stopwatch when the cart stops.

**4.** Record the time.

**5.** Move the magnets up by about 1 mm to make the system under-damped.

**6.** Repeat steps 1 through 4.

**7.** Move the magnets down as close to the track as you can get them (without touching) to make the system over-damped.

**8.** Repeat steps 1 through 4.

*Skip to the Question on page 7.*

## Method II (using a motion sensor)

### Sensor, Interface, and Software Set-up

*For detailed information about setting up your motion sensor, interface, and software, refer to the instructions supplied with those products.*

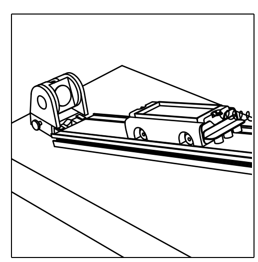
**1.** Place a motion sensor at the lower end of the track.

**2.** Connect the sensor to your interface.

**3.** Set the sampling rate to 50 Hz.

**4.** Prepare a graph to display Position versus time.

**5.** Take some test data and adjust the sensor so that it can measure the cart's position up to a distance of about 70 cm. Delete the test data.



**Motion sensor at end of track**

### Data Collection

1. Pull the cart up the track *exactly* to the mark.
2. Release the cart and start data recording at the same time.
3. Wait until the cart stops.
4. Stop data recording.
5. Move the magnets up by about 1 mm to make the system under-damped.
6. Repeat steps 1 through 4.
7. Move the magnets down as close to the track as you can get them (without touching) to make the system over-damped.
8. Repeat steps 1 through 4.

## Question

What type of damping (under, over, or critical) makes the cart stop in the shortest time?

## Further Study: What factors affect critical damping?

You have already discovered that reducing the height of the magnets can change the system from under-damped to critically damped to over-damped. What other factors can you adjust to change the system's damping behavior?

Starting with a critically damped system each time, make the following changes to the system and observe the result.

- Release the cart from a different position on the track.
- Add about 20 g of mass to the cart.
- Change the angle of the track.
- Change the spring constant (replace or remove one of the springs).

## Further Study: Theoretical model

For the theoretical model of the cart's position as a function of time, we will assume that the spring is massless and obeys Hooke's law; that the damping force is proportional to velocity; and that there are no forces acting on the cart other than the spring force, the damping force, the force of gravity, and the normal force of the track. We will also set the condition that the initial velocity $v(0) = 0$ at time $t = 0$. The motion of the cart is determined with the following quantities:

$x$, the position of the cart with $x = 0$ at the equilibrium position

$x_0$, the initial position at time $t = 0$

$m$, the mass of the cart

$k$, the spring constant ($F_{spring} = -kx$)

$b$, the damping constant ($F_{damping} = -bv$) determined by the height of the magnets

$\omega_0 = \sqrt{k/m}$, the natural frequency of the system in the absence of damping

$\gamma = b/m$, a quantity with units of frequency

If $m$ and $k$ are held constant, then the damping behavior of the system (whether it is under-, over-, or critically damped) depends on the value of $b$.

### Under damping

For the system to be under-damped, we must have a relatively low value of $\gamma$. Let $\gamma = \gamma_u$ such that $\omega_0^2 - \gamma_u^2/4 > 0$. Then

$$x(t) = x_0 e^{-\gamma_u t/2}\left[\cos(\omega t) + \frac{\gamma_u}{2\omega}\sin(\omega t)\right]$$

Where $\omega = \sqrt{\omega_0^2 - \gamma_u^2/4}$.

### Over damping

For over damping, the value of $\gamma$ must be higher. Let $\gamma = \gamma_o$ such that $\omega_0^2 - \gamma_o^2/4 < 0$. Then

$$x(t) = x_0\left(\frac{1}{2} - \frac{\gamma_o}{4\beta}\right)e^{-(\gamma_o/2 + \beta)t} + x_0\left(\frac{1}{2} + \frac{\gamma_o}{4\beta}\right)e^{-(\gamma_o/2 - \beta)t}$$

Where $\beta = \sqrt{\gamma_o^2/4 - \omega_0^2}$.

### Critical damping

Critical damping occurs at a specific value of $\gamma$. Let $\gamma = \gamma_c$ such that $\omega_0^2 - \gamma_c^2/4 = 0$. Then

$$x(t) = x_0\left(1 + \frac{\gamma_c t}{2}\right)e^{-\gamma_c t/2}$$

### Modeling the cart's motion

Graph the three equations above and compare them to your actual data. Either measure or estimate your experimental values of $x_0$, $m$, and $k$ (or $\omega_0$) to put into the equations. Estimate values of $b$ for under, over, and critical damping.

# Experiment 3: Predicting Terminal Velocity

| Equipment | Part Number |
|---|---|
| Dynamics Cart Magnetic Damping | ME-6828 |
| Motion Sensor | PS-2103A *or similar* |
| Cart[1] | ME-6950, ME-9430, *or* ME-9454 |
| Aluminum Track[1] | ME-6953 *or similar* |
| IDS Pivot Clamp[1] | ME-9810 |
| Large Table Clamp | ME-9472 |
| 90 cm Rod | ME-8738 *or similar* |
| Angle Indicator[2] | ME-9495 |

*[1]These items are components of many PASCO dynamics systems.*
*[2]Or other way to measure track angle*

## Introduction

In this experiment, you will study the motion of a magnetically damped cart. First, you will discover the relationship between the velocity and acceleration as it comes to a stop after being pushed on a level track. Using this relationship, you will predict the cart's terminal velocity on a inclined track. Finally, you will test your prediction.

## Part I: Acceleration versus Velocity

### Theory

The magnetic drag force, $f_m$, is proportional to (and in the opposite direction of) velocity $v$

*(eq. 1)* $$f_m = -bv$$

where $b$ is a positive constant.

On a level track, assuming no other sources of friction, $f_m$ is the only force (along the axis of movement) acting on the cart. Therefore, by Newton's second law

*(eq. 2)* $$f_m = ma$$

where $m$ is the mass of the cart and $a$ is the acceleration. (Note that $f_m$ and $a$ are not necessarily constant over time.)

The combination of equations 1 and 2 gives the relationship between $a$ and $v$

*(eq. 3)* $$v = -\frac{m}{b}a$$

In a graph of $v$ versus $a$, the slope will equal $-m/b$ .

Equation 3 assumes that magnetic drag is the only source of friction. A real cart is also affected by friction that is not proportional velocity, but is constant as long as the cart is moving. If this additional frictional force is $f_0$, then the net force on the cart is

*(eq. 4)* $$F_{net} = ma = -bv - f_0 \qquad (\text{for } v > 0)$$

and

*(eq. 5)* $$v = -\frac{m}{b}a - \frac{f_0}{b}$$

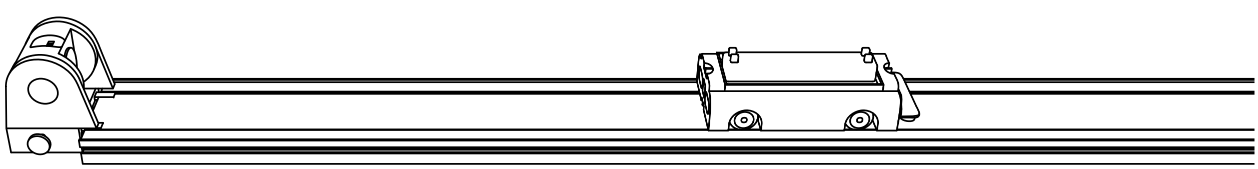
The addition of $f_0$ does affect the slope of the $v$ versus $a$ graph, but it does add a vertical offset (or "Y-intercept') equal to $-f_0/b$ .

In this part of the experiment, you will use the motion sensor to record $v$ and $a$ as the cart slows down after you push it on a level track.

### Set-up

1. Make the track level.
2. Attach a motion sensor to the left end of the track.
3. Attach the Magnetic Damping Accessory to the cart.
4. If you are using a plastic cart, add about 250 g of mass, so that the total mass is about 500 g. (Additional mass is not necessary with a metal cart.)
5. Place the cart on the track with the Magnetic Damping Accessory away from the motion sensor.
6. Adjust the magnets to be about 1 mm above the track. Check to make sure that the magnets do not touch the track when you push the cart swiftly.

*Important: After you have adjusted the magnets, do not readjust them for the remainder of the experiment.*



**Experiment set-up for Terminal Velocity, Part I**

### Sensor, Interface, and Software Set-up

*For detailed information about setting up your motion sensor, interface, and software, refer to the instructions supplied with those products.*

1. Connect the sensor to your interface.
2. Prepare a graph (or graphs) to display Position, Velocity, and Acceleration versus time.
3. Take some test data and adjust the sensor so that it can measure the cart's position up to a distance of about 1 m. Delete the test data.

*Leave the sample rate at the motion sensor's default of 10 Hz.*

### Data Collection

You may need to try several times to complete the following steps successfully. If you do not get it right, delete your data and try again. The recorded data should show the motion of the cart only *after* you release it and *before* it comes to a complete stop. There should be about 1 s of recorded data. Use one hand to push the cart and the other hand to start and stop data recording.

1. Using a smooth, sweeping motion, push the cart away from the motion sensor and release it. Make sure that your hand does not prevent the motion sensor from detecting the cart.

2. About 0.1 s after releasing the cart, start data recording.

3. Just before the cart stops, stop data recording.

### Analysis

1. Create a graph showing velocity (on the vertical axis) versus acceleration (on the horizontal axis).

2. Apply a linear fit to the data.

According to equation 5, the slope of the best-fit line equals $-m/b$ and the Y-intercept equals $-f_0/b$.

## Part II: Predicting Terminal Velocity

### Theory

If a cart is allowed to roll down an inclined track, its velocity will increase until the frictional forces acting against the direction of movement equal the gravitational force acting in the direction of movement. At that point, the net force on the cart is zero, the acceleration is zero, and the velocity remains constant. This velocity is known as the terminal velocity, $v_T$.

The gravitational force acting on the cart (in the direction of movement) is $mg\sin\theta$, where $g$ = 9.81 m/s$^2$ and θ is the angle of incline. At terminal velocity, the net force is

*(eq. 6)*
$$F_{net} = 0 = mg\sin\theta - bv_T - f_0 \qquad (\text{for } v_T > 0)$$

The values of $b$ and $f_0$ are the same as they were on the level track.

Solving equation 6 gives

*(eq. 7)*
$$v_T = \frac{m}{b}g\sin\theta - \frac{f_0}{b}$$

### Analysis

In Part 1, you found the values of $-m/b$ and $-f_0/b$. Use these values and equation 7 to predict the terminal velocity of your cart on a track inclined at θ = 3.0°.

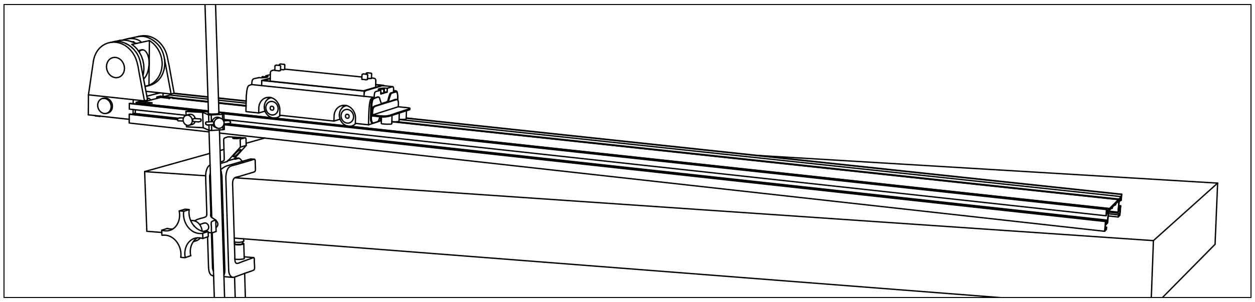
At θ = 3.0°, $v_T$ = _________________, prediction

## Part III: Measuring Terminal Velocity

### Set-up

**1.** Take the cart off the track and set it aside. (Be careful not to let the Magnetic Damping Accessory shift on the cart.)

**2.** Use a table clamp, rod and pivot clamp to raise the left end of the track (with the motion sensor attached). Adjust the incline to 3.0°.



**Experiment set-up for Terminal Velocity, Part III**

### Data Collection

**1.** Place the cart back on the track and hold it about 15 cm from the motion sensor with the magnets away from the sensor.

*Note: Before releasing the cart, be prepared to stop it so that the Magnetic Damping Accessory is not knocked out of position.*

**2.** Start data recording.

**3.** Release the cart, stop it after it has rolled about 1 m.

**4.** Stop data recording.

### Analysis

From your recorded data, determine the terminal velocity.

At $\theta = 3.0°$, $v_T$ = _________________, measured

## Questions

**1.** How close was your prediction to your measured value of $v_T$? If they were not exactly equal, what might account for the difference?

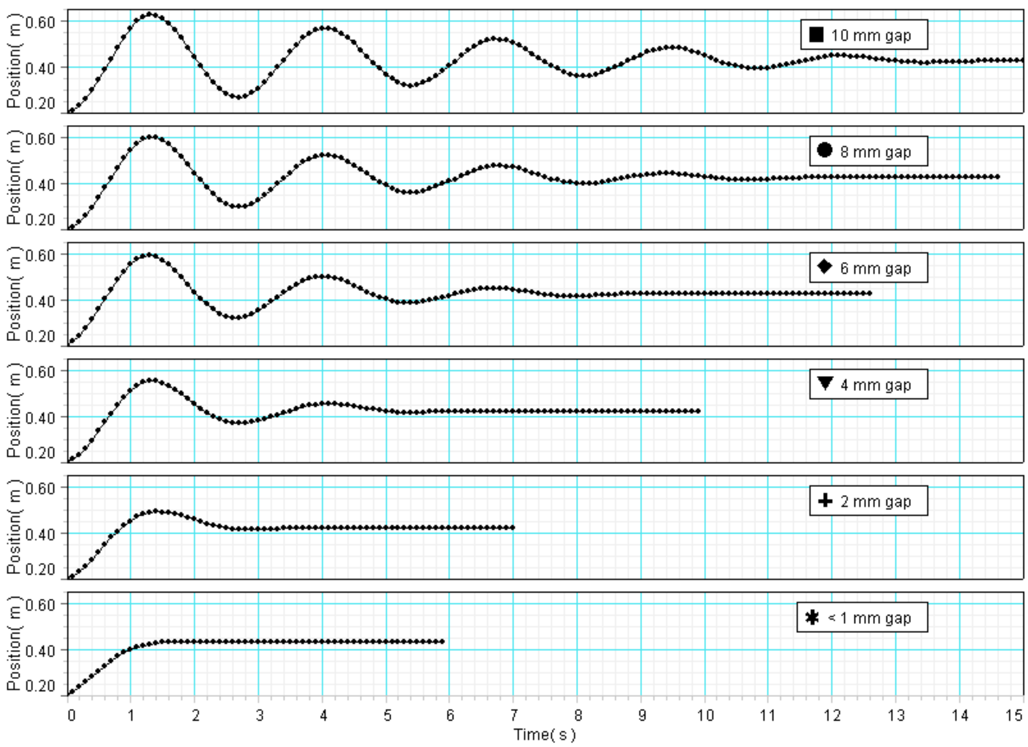
**2.** Would the terminal velocity change if you changed the mass of the cart?

**3.** Did you have to measure the mass of the cart to predict the terminal velocity?

# Sample Data

## Experiment 1: Damped Oscillation

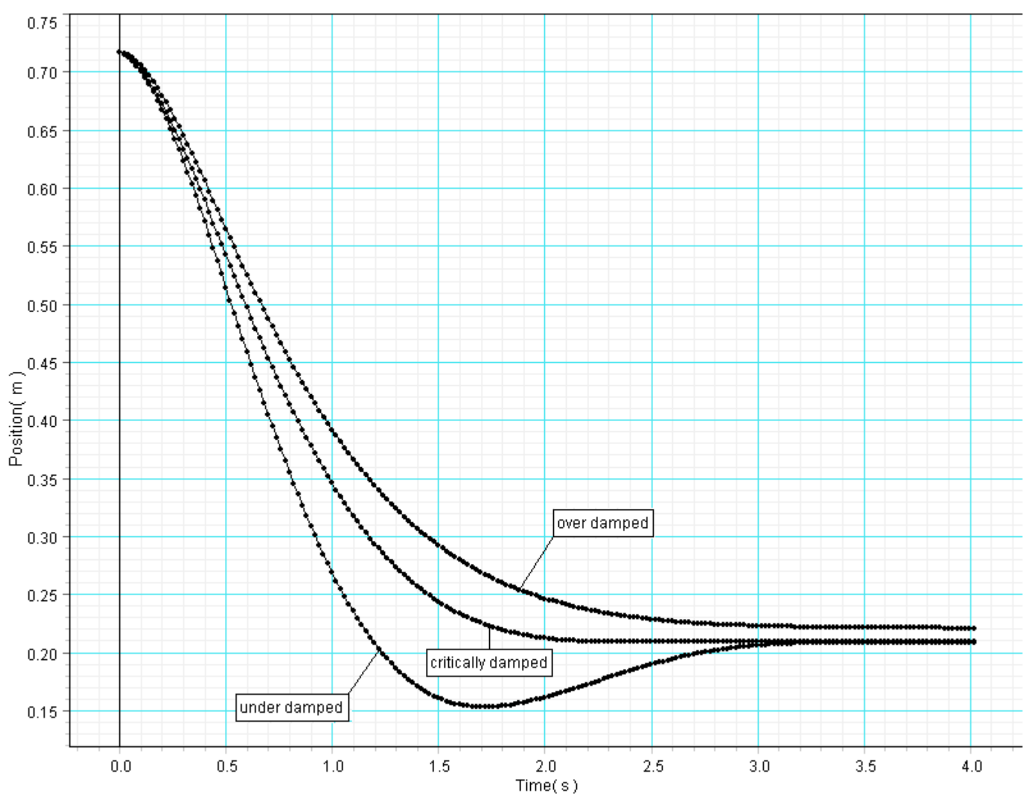
Damping increases as the magnets are moved closer to the track.




## Experiment 2: Critical Damping

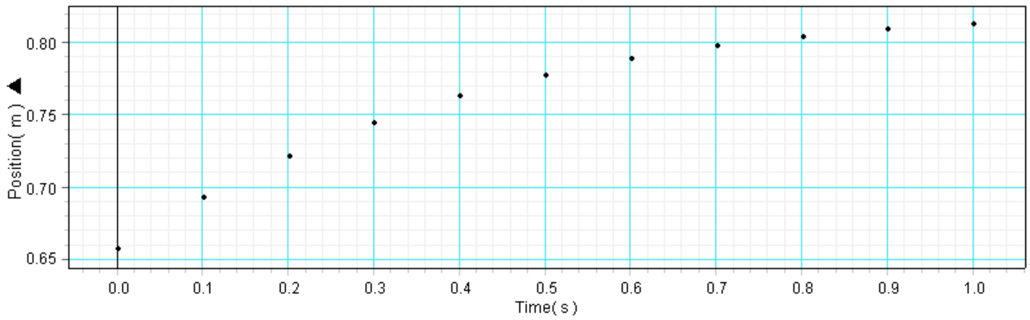
Critical damping makes the cart stop in the shortest time.

Note that, due to static friction, the over-damped cart stops at a slightly different position.

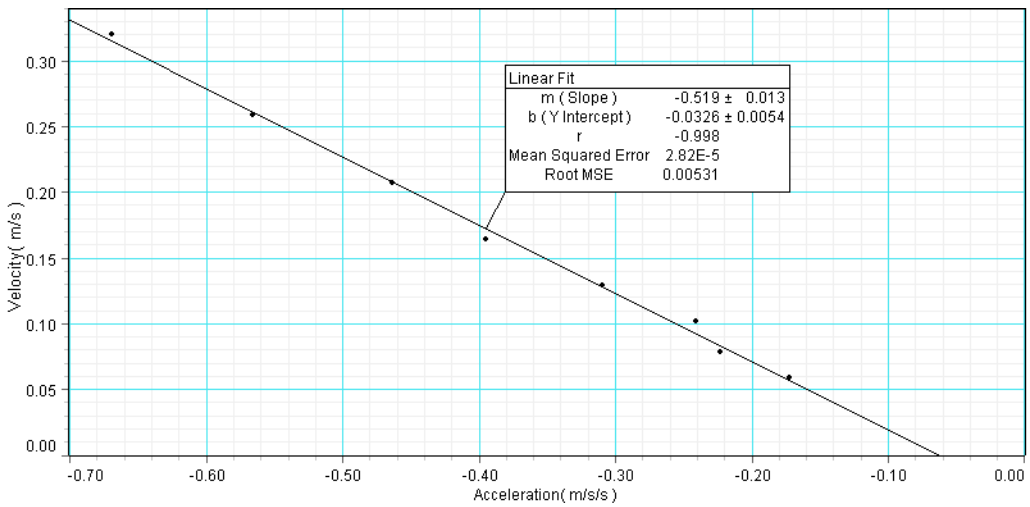


## Experiment 3: Predicting Terminal Velocity

The graph below shows position versus time as the cart is coming to a stop on a level track.

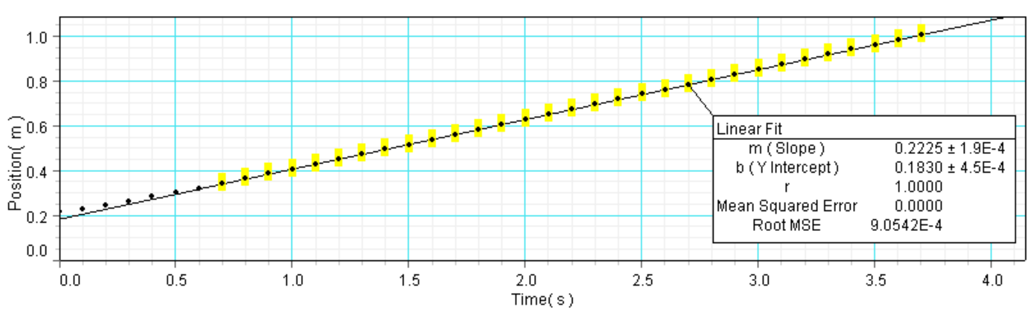



The graph below shows velocity versus acceleration for the cart on the level track.




From the slope and intercept of the best-fit line: $-m/b = -0.52\text{ s}$ and $-f_0/b = -0.033\text{ m/s}$. From these values and equation 7, the *predicted* terminal velocity is $v_T = 0.23\text{ m/s}$.

The graph below shows position versus time for the cart running down a 3° incline. The slope of the linear portion is the *measured* terminal velocity: $v_T = 0.2225\text{ m/s}$.




In this case the predicted and measured values differ by about 0.01 m/s, or 4%. The difference may be due to uncertainty in the values of $m/b$, $f_0/b$, and the angle of the inclined track.

The terminal velocity would be different if the cart's mass were changed. In this experiment, it is not necessary to know the mass of the cart to predict the terminal velocity.

# Technical Support

Before you call PASCO technical support, have the apparatus and this user's guide available. Please note the following:

- Product name and model number (e.g., Large Structures Set, ME-7003)
- Approximate age of the product;
- Detailed description of the problem/sequence of events required to duplicate the problem.

For assistance with any PASCO product, contact PASCO at:

Address: PASCO scientific
10101 Foothills Blvd.
Roseville, CA 95747-7100

Phone: +1 916 462 8384 (worldwide)
877-373-0300 (U.S.)

Web: www.pasco.com

Email: support@pasco.com

**Limited Warranty**
For a description of the product warranty, see the PASCO catalog.