Projectile Motion

EQUIPMENT

<table>
<thead>
<tr>
<th>Item</th>
<th>Code</th>
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<tbody>
<tr>
<td>Mini Launcher</td>
<td>ME-6825</td>
</tr>
<tr>
<td>Time of Flight Accessory</td>
<td>ME-6810</td>
</tr>
<tr>
<td>Phone Jack Extender Cable</td>
<td>PI-8117</td>
</tr>
<tr>
<td>Photogate Head</td>
<td>ME-9498A</td>
</tr>
<tr>
<td>Photogate Bracket</td>
<td>ME-6821A</td>
</tr>
<tr>
<td>Large C Clamp (need one only)</td>
<td>SE-7285</td>
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<tr>
<td>Plumb Bob (need one only)</td>
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</tr>
<tr>
<td>Carbon Paper</td>
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<tr>
<td>Metric Measuring Tape</td>
<td>SE-8712A</td>
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<tr>
<td>850 Universal Interface</td>
<td>UI-5000</td>
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<tr>
<td>PASCO Capstone</td>
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NOT INCLUDED, BUT REQUIRED:

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INTRODUCTION

The purpose of this experiment is to predict the horizontal range of a projectile shot from various heights and angles. In addition, students will compare the time of flight for projectiles shot horizontally at different muzzle velocities. This is really four stand-alone activities except that all require a measurement of initial speed when the launcher is fired horizontally. This is done in Procedure 1, but is easy to redo. Doing all four is too long for a 3 hour lab, but Procedure 1 and 2, or Procedure 1 and 3 should work well together. Procedure 1 deals with the independence of horizontal and vertical motion and is a key idea that should not be skipped. Procedure 4 is short but requires knowledge of Calculus.

Uncertainty

The launchers work extremely well and with a combination of luck and skill you may see an accuracy of a centimeter or so over a 3-meter distance. However, small variations in cocking the launcher may cause the pattern to shift by several centimeters. We use three shot patterns and average and the results should generally be accurate to within 3 centimeters.
THEORY

The horizontal range, \( \Delta x \), for a projectile can be found using the following equation:

\[ \Delta x = v_x t \quad (1) \]

where \( v_x \) is the horizontal velocity (= the initial horizontal velocity) and \( t \) is the time of flight.

To find the time of flight, \( t \), the following kinematic equation is needed:

\[ \Delta y = \frac{1}{2} a_y t^2 + v_{y0} t \quad (2) \]

where \( \Delta y \) is the height, \( a_y = -g \) is the acceleration due to gravity and \( v_{y0} \) is the vertical component of the initial velocity.

When a projectile is fired horizontally (from a height \( \Delta y \)), the time of flight can be found by rearranging Equation 2. Since the initial vertical velocity is zero, the last term drops out of the equation yielding:

\[ t = \left( \frac{2\Delta y}{a_y} \right)^{1/2} = \left( -\frac{2\Delta y}{g} \right)^{1/2} \quad (2a) \]

When a projectile is fired at an angle and it lands at the same elevation from which it was launched, \( \Delta y = 0 \), and we may solve Equation (2) for \( t \):

\[ t = \frac{2v_{y0}}{g} \quad (2b) \]

Substituting this into Equation (1) yields

\[ \Delta x = 2v_x v_{y0}/g = \frac{(2v^2 \cos \theta \sin \theta)}{g} \quad (3) \]

where \( v \) is the initial speed of the projectile.

When a projectile is fired from a height, none of the terms drop out and Equation 2 may be rearranged as follows:

\[ \frac{1}{2} a_y t^2 + v_{y0} t - \Delta y = 0 \quad (2c) \]

Equation 2c may be solved using the quadratic formula to find the time of flight, \( t \). Equation 1 then yields the horizontal range.
SETUP 1 – Muzzle Velocity, Time of Flight, and Range

**Safety**
Wear Safety Goggles.  
Do not place foreign objects into the Launcher.  
Do not look into the Launcher.  
Do not aim the Launcher at others.

1. Slide the Photogate Bracket into the groove on the bottom of the launcher and tighten the thumbscrew.

2. Connect two photogates to the bracket (see photos below). Adjust the Photogate Bracket so the first photogate is as close to the launcher as possible without blocking the IR beam.

3. Attach the launcher to the launcher stand using the upper holes (see photo below).

4. Plug the photogate closest to the launcher into Digital Input 1 on the 850 Universal Interface. Plug the other photogate into Digital Input 2.


6. In PASCO Capstone, click on Digital Input 1 in the Hardware Setup and select a photogate. Then click on Digital Input 2 and select another photogate. Then click on Digital Input 3 and select a Time-of-Flight Accessory.
7. Open the Timer Setup and select a pre-configured timer for Time-of-Flight with Two Photogates as shown below:

![Diagram of the selected timer setup as shown below](image)

8. Choose one corner of a table to place the projectile launcher. Make sure a distance of about 3 meters is clear on the floor in the direction the ball will be fired.
9. Clamp the launcher to the corner of the table using a C-clamp (see photo below).

10. Using the attached plumb bob, adjust the angle of the launcher to 0°.

Shooting Horizontally onto the Floor

**Note 1: Measuring angles**

It is critical that you measure the angle carefully. An error of ½ degree will affect your results by several centimeters. You should be able to read the angles within 0.2 degrees. In Figure 4, the angle is 30.0°. Does Figure 5 show 30.0°? No! Notice that the line is not symmetric between the 29° and 31° marks. This angle should be read as 29.8°. In Procedure 1, it is critical that the angle be exactly 0.0°. In Procedures 2 & 3, it is not critical to exactly set the requested angle as long as you know what the exact angle is. For example, having 20.7° instead of 20.0° is fine as long as you know it is 20.7° and adjust the numbers in the table to reflect your actual angles.
PROCEDURE 1: Muzzle Velocity, Time of Flight, and Range

Note the circle on the side of the launcher that says “Launch Position of Ball.” This indicates the position of the ball when it leaves the spring and becomes a free projectile. From where on the circle should you measure the drop distance $\Delta y$? Hint: what part of the ball strikes the floor? Is $\Delta y$ positive or negative? From what part of the circle should you measure $\Delta x$? In addition, measure the spacing between the two photogates and verify that it is 10.0 cm. The program calculates the initial speed by assuming the photogates are separated by 10.0 cm and dividing by the time the ball takes to travel between the two gates.

1. Measure the drop distance, $\Delta y$, to the top of the Time of Flight Accessory.

2. Carefully adjust the launcher to fire horizontally. The protractor should read exactly zero degrees. You should try to set the angle within 0.2 degrees by making the string equidistant between the +1 and -1 degree hash marks on the protractor. You will not get good results if you are not careful when setting the angles. See Note 1.

3. Place the steel ball into the launcher and use the push rod to load the ball until the third “click” is heard.

4. Check to see that there is no one down range! Launch the ball by pulling straight upward on the string. Don’t jerk. Observe where the ball hits the floor. Tape a small piece of tape to the floor to mark the spot. Place the Time of Flight Accessory above the piece of tape so the ball will strike it.

5. Reload the ball to the third “click” and start recording in Capstone.

6. Pull the launch cord on the launcher. Click the Stop button to stop recording.

7. Create a table to record the Initial Speed and the Time of Flight as shown below. Create User-Entered data sets called “# Clicks” with no units, “V initial” with units of m/s, and “Time-of-Flight” with units of seconds.

<table>
<thead>
<tr>
<th>Table 1: Time of Flight vs. Initial Speed</th>
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<tbody>
<tr>
<td># Clicks</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
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<td>4</td>
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<td>7</td>
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<tr>
<td>8</td>
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<tr>
<td>9</td>
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</tbody>
</table>
8. Repeat two more times with the launcher cocked to three clicks.

9. If the launcher is only compressed to two clicks, will the time of flight be more, less, or the same as for 3 clicks? Explain your logic.

10. Repeat steps 2-8 for 2 clicks and 1 click. Record your results in Table I. Did you answer correctly in step 9? Why or why not?

11. Using your data and Equation 1, calculate the $\Delta x$ distance to where the ball should strike the floor when the launcher is compressed two clicks. Drop a plumb line from the center of the Launch Position of Ball circle and mark the position with a piece of tape. Measure a distance equal to $\Delta x$ along a line between the piece of tape you just put on the floor and the one from step 4. Tape a piece of white paper to the Time of Flight apparatus and place it at your predicted impact point. Mark your predicted impact point with an X. Place a piece of carbon paper on top of the paper (face down). Launch the ball three times using the two click position. Turn in this paper as part of your lab report!

**PART 1 CONCLUSIONS**

1. Did the time of flight depend on the initial horizontal speed. What does this imply about the dependence of the vertical motion on the horizontal motion?

2. Use Equation 2a from the theory to calculate the time of flight.

3. How well does the time of flight calculated from Equation 2a agree with your experimental values. If they don’t agree, what could explain the difference?

4. How well did your predicted range compare to the actual range? What does this show?

5. How would the horizontal range change if the muzzle velocity was doubled? Explain how you know.

6. How would the horizontal range change if the height from the ground was doubled? Explain how you know.

7. How would the horizontal range change if the mass of the ball was doubled? Explain how you know.

8. What effect are we able to ignore in this experiment? Explain.
PROCEDURE 2: Launching at an Angle on a Plane

1. Create a table as shown below. Create User-Entered data sets called “Ang” with units of degrees, “V1” with units of m/s, and “MRange” (Measured Range) with units of meters. Then create two calculations:

\[ V = \left( [V1]^2 - 2 \times [\text{Accel due to gravity (g)}] \times 0.035 \times \sin([\text{Ang}]) \right)^{0.5} \]

with units of m/s

**Predicted Range (PRange)**

\[ \text{PRange} = 2 \times [V]^2 \times \cos([\text{Ang}]) \times \sin([\text{Ang}]) / [\text{Accel due to gravity (g)}] \]

with units of m

<table>
<thead>
<tr>
<th>Table II: Plane Range</th>
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<tbody>
<tr>
<td>Ang (°)</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>25</td>
</tr>
<tr>
<td>35</td>
</tr>
<tr>
<td>45</td>
</tr>
<tr>
<td>55</td>
</tr>
<tr>
<td>65</td>
</tr>
</tbody>
</table>
2. If you did Procedure 1, calculate the average initial speed in meters per second for 1-click horizontal fire. Record the average value in the second column (V1) of Table II. If you are doing this part of the experiment without doing Procedure 1, you will have to determine the speed for horizontal fire by performing Procedure 1, steps 3-8. You do not need the Time of Flight accessory.

3. Clamp the launcher to the edge of a table using a C-clamp so that the ball launches from and lands at the same elevation (the bottom of the Ball Launch Position circle should be even with the top of the table (see photos above). Launcher should be as far back as possible on its track so the front holding screw points directly at the center of the Launch Position circle. In this manner, the release height will not change when you change the angle.

4. Adjust the launcher for a launch angle of 45°. Using the push rod, push the ball into the Launcher until the first click is heard. Using the string, pull back on the trigger. Note the location on the table where the ball lands.

5. Tape a sheet of blank paper at the location where the ball landed. Place carbon paper over the blank paper.

6. Load the Launcher to the first click again. Launch the ball. Repeat two more times.

7. Use the tape measure to find the horizontal range from the Ball Launch Position circle to the center of the three shot pattern (just estimate the center of the pattern).

8. Record the value of the horizontal range in meters into the Measured Range (MRange) column of Table II. If your angle was not exactly 45°, record the correct angle in the Angle (Ang) column.

9. Repeat the steps 4-8 for 25, 35, 55, 65 degrees.

ANALYSIS 2

1. The third column in Table II shows the initial speeds (V₀) at various angles calculated from your measured initial horizontal speed. They are smaller because when the gun is tipped up by an angle θ, some of the energy from the spring goes into increasing the potential energy of the system instead of the kinetic energy of the ball. However, since you have probably not considered energy yet in your physics class, we have calculated the speeds for you. The formula is:

   \[ v^2 = (v_0)^2 - 2gs \sin \theta \]

   where \( v \) = initial speed, \( v_0 \) = horizontal speed (measured), \( s \) = distance spring is compressed and \( \theta \) is the angle of tip. \( s = 3.5 \text{ cm for 1 click}, \ 4.8 \text{ cm for two clicks}, \ 6.3 \text{ cm for three clicks} \).

2. We now use Equation 3 to calculate the predicted range (PRange).
3. Compare your measured values to the predicted values for the range. Do they agree? Try to explain any differences.

4. Create a graph of MRange vs. Ang. Then use “Add the similar measurement” to add PRange to the vertical axis.

PART 2 CONCLUSIONS

1. Are the plots for the Measured Range vs. Angle and the Predicted Range vs. Angle the same? Apply a quadratic fit to each. Try to explain any difference.

2. Refer to your Range vs. Angle graph. What angle corresponds to the maximum range? Why isn’t the graph symmetric about 45°?
PROCEDURE 3: Launching at an Angle from a Height

1. Create a table as shown below. Create User-Entered data sets called “Angle” with units of degrees, “V3” with units of m/s, and “ERange” (Experimental Range) with units of meters. Then create calculations:

   **Corrected velocity (Vcor)**
   \[ V_{cor} = \left( V_3^2 - 2 \times \text{Accel due to gravity (g)} \times 0.063 \times \sin(\text{Angle}) \right)^{0.5} \]
   with units of m/s

   **Theoretical Time of Flight (Tflight)**
   \[ T_{flight} = \left( V_{cor} \times \sin(\text{Angle}) + \left( V_{cor} \times \sin(\text{Angle}) \right)^2 - 2 \times \Delta y \times \text{Accel due to gravity (g)} \right)^{0.5}/\text{Accel due to gravity (g)} \]
   with units of seconds, and \( \Delta y \) is the change in height of the ball.

   **Theoretical Range (TRange)**
   \[ T_{Range} = V_{cor} \times \cos(\text{Angle}) \times T_{flight} \]
   with units of meters.

<table>
<thead>
<tr>
<th>Table III: Range vs. Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle (^{\circ})</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>-20</td>
</tr>
<tr>
<td>-10</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>10</td>
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<td>45</td>
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<tr>
<td>50</td>
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<tr>
<td>60</td>
</tr>
</tbody>
</table>
2. If you did Procedure 1, calculate the average initial speed in meters per second for 3-click horizontal fire, \( V_{\text{initial}} \). Record the average value in the second column (\( V_3 \)) of Table III. If you are doing this part of the experiment without doing Procedure 1, you will have to determine the speed for horizontal fire by performing Procedure 1, steps 3-8 for the 3-click position. You do not need the Time of Flight accessory.

3. Clamp the launcher to the edge of a table using the C-clamp so that the ball launches from a position above where it will land (see photo above). Launcher should be as far back as possible on its track so the front holding screw points directly at the center of the Launch Position circle. That way the release height will not change when you change the angle.

4. Adjust the angle of the launcher to \(-20^\circ\).

5. Note the circle on the side of the launcher that says “Launch Position of Ball.” This indicates the position of the ball when it leaves the spring and becomes a free projectile. From where on the circle should you measure the drop distance \( \Delta y \)? Hint: what part of the ball strikes the floor? Is \( \Delta y \) positive or negative? From what part of the circle should you measure \( \Delta x \)? Measure the drop distance in meters. Click open the Calculator in Capstone and type in the value for \( \Delta y \).

6. Drop a plumb line from the Launch Position of Ball circle and mark the position with a piece of tape. Measure the distance \( \Delta x \) from this mark for each angle.

7. Using the plunger, push the ball as far as possible into the Launcher. Make sure three clicks are heard. Using the string, pull back on the trigger. Keep track of the location on the floor where the ball lands.

8. Tape a sheet of blank paper at this location. Place carbon paper over the blank paper.

9. Load the Launcher. Take 3 shots. Measure \( \Delta x \) to the center (eyeball it) of the three shot pattern with the tape measure and record it in the Experimental Range column of Table III on the Analysis 3 page. If your angle was not exactly 45\(^\circ\), record the correct angle in the Angle column.

10. Repeat for angle of \(-10^\circ, 0^\circ, 10^\circ, 20^\circ, 30^\circ, 35^\circ, 40^\circ, 45^\circ, 50^\circ, \) and \(60^\circ\).

**ANALYSIS 3: Launching at an Angle from a Height**

1. See the discussion of why the initial launch speed \( V_3 \) is different than the horizontal launch speed under the Analysis 2 section.

2. We now use Equation 2c from Theory to calculate the time of flight. Solve the quadratic equation for \( t \). Verify that the calculations for \( T_{\text{flight}} \) (the time of flight) agree with your result.
3. Verify that the calculations for the Theory Range (TRange) agree with Equation 1.

4. Compare your measured values to the predicted values. Do they agree? Try to explain any differences.
5. Create a graph of TRange vs. Ang. Then use “Add the similar measurement” to add ERange to the vertical axis.

PART 3 CONCLUSIONS

Examine the graph of Experimental Horizontal Range vs. Angle and the Theory Horizontal Range vs. Angle. How well do the measured results compare to the predictions? What does this show? Explain any differences.
PROCEDURE 4: Challenge Activity – Maximum Height of a Projectile

THEORY 4

We now consider the problem of a launcher at a fixed distance from a vertical wall and seek to find the angle that causes the projectile to strike the wall at a maximum height. In this case, solving equation 2, gives the time to the wall as

\[ t = \frac{R}{v_{ox}} \]  \hspace{1cm} (4)

where \( R \) is the distance to the wall. Then the height (\( h \)) at the wall is given by

\[ h = v_{oy}t - \frac{1}{2} gt^2 = \frac{v_{oy}R}{v_{ox}} - \frac{1}{2} g\left(\frac{R}{v_{ox}}\right)^2 = \frac{R(v_o \sin\theta)}{v_o \cos\theta} \frac{R}{2v_o^2} \frac{R}{\cos^2\theta} \]

\[ h = R \tan\theta - \frac{gR^2}{2v^2} \cos^2\theta \]  \hspace{1cm} (5)

Use calculus to find the angle (call it “max angle”) that maximizes the height.

SETUP 4: Launching at a Vertical Wall

1. Clamp the launcher to the edge of a table using the C Clamp about 1.5 meters from a vertical wall. Launcher should be as far back as possible on its track so the front holding screw points directly at the center of the Launch Position circle. That way the release height will not change when you change the angle.
PROCEDURE 4

1. Measure the distance from the Launch Position circle (what portion of the circle should you use?) to the wall. Record this value.

2. You will need to measure the horizontal speed for the two-click position if you don’t already have it. See procedure 1. As before in Procedure 3, we correct this value assuming the correct angle will be around 45°.

   \[ V_{corrected} = \left( \frac{V^2}{2} - 2 \times \text{Accel due to gravity (g)} \times 0.048 \times \sin(45) \right)^{0.5} \]

3. Use the expression that you derived in the theory to calculate the theoretical angle that gives the maximum height.

4. Adjust the launcher to the theoretical angle for maximum height. Using the push rod, push the ball into the Launcher until the second click is heard. Using the string, pull back on the trigger. Note the location on the wall where the ball strikes.

5. Tape a sheet of blank paper at the location where the ball struck. The location where the ball hit should be near the top of the paper. Tape carbon paper over the blank paper.

6. Fire three shots at the wall. Circle the pattern and label each spot “max”. Leave the paper in place.

7. Now increase the launch angle by five degrees. Launch a three group pattern. Label each spot as max + 5.

8. Now decrease the launch angle to five degrees less than Max Angle. Launch a three group pattern. Label each spot as max - 5.

9. If the +/- 5 patterns are below the max patterns, you are finished.

ANALYSIS 4: Maximum Height

Briefly discuss your results.