Capacitance

Equipment

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Description</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Basic Electrometer</td>
<td>ES-9078</td>
</tr>
<tr>
<td>1</td>
<td>Basic Variable Capacitor</td>
<td>ES-9079</td>
</tr>
<tr>
<td>1</td>
<td>Electrostatics Voltage Source</td>
<td>ES-9077</td>
</tr>
<tr>
<td>1</td>
<td>Short Patch Cords (set of 8)</td>
<td>SE-7123</td>
</tr>
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</table>

Required but not included:

<table>
<thead>
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<th>Description</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>850 Universal Interface</td>
<td>UI-5000</td>
</tr>
<tr>
<td>1</td>
<td>PASCO Capstone</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Paper</td>
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Introduction

The purpose of this experiment is to investigate how the capacitance of a parallel-plate capacitor varies when the plate separation is changed and to qualitatively see the effect of introducing a dielectric material between the plates. A computer model of the system will be developed and the student will observe some of the power of computer modeling.

Theory

A capacitor is used to store charge. A capacitor can be made with any two conductors kept insulated from each other. If the conductors are connected to a potential difference, $V$, as in for example the opposite terminals of a battery, then the two conductors are charged with equal but opposite amount of charge $Q$, which is then referred to as the “charge in the capacitor.” The actual net charge on the capacitor is zero. The capacitance of the device is defined as the amount of charge $Q$ stored in each conductor after a potential difference $V$ is applied:

$$ C = \frac{Q}{V} $$

Rearranging gives:

$$ V = \frac{Q}{C} \quad (1) $$

The simplest form of a capacitor consists of two parallel conducting plates, each with area $A$, separated by a distance $d$. The charge is uniformly distributed on the surface of the plates. The capacitance of the parallel-plate capacitor is given by:

$$ C = \kappa \varepsilon_0 \frac{A}{d} $$

Where $\kappa$ is the dielectric constant of the insulating material between the plates ($\kappa = 1$ for a vacuum; other values are measured experimentally and can be found in tables), and $\varepsilon_0$ is the
permittivity constant, of universal value \( \varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m} \). The SI unit of capacitance is the Farad (F).

The system we use is more complex. In addition to the two moveable parallel plates, the connecting wires and the electrometer also have some capacitance. This capacitance is roughly equal to the capacitance of the moveable plates when the plates are 1 cm apart and cannot be ignored. Including this gives:

\[
C = \kappa \varepsilon_0 A/d + C_{\text{sys}}
\]  

(2)

where \( C_{\text{sys}} \) is the capacitance of the rest of the system. Substitution of Equation 2 into Equation 1 yields:

\[
V = Q/(\kappa \varepsilon_0 A/d + C_{\text{sys}})
\]  

(3)

Any material placed between the plates of a capacitor will increase its capacitance by a factor \( \kappa \) called the dielectric constant where:

\[
C = \kappa C_0
\]  

(4)

with \( C_0 \) being the capacitance when there is a vacuum between the plates of the capacitor. Dielectric materials are non-conductive. Any dielectric material can be used to keep the plates in a capacitor insulated from each other (preventing them from touching and discharging). To three significant figures, \( \kappa = 1.00 \) for air. For all materials, \( \kappa > 1 \). If the charge on a capacitor is kept constant while a dielectric is inserted between the plates, Equations 1 & 4 yield:

\[
Q = CV = C_0 V_0 = C/(\kappa \varepsilon_0 V_0)
\]  

so

\[
V = V_0/\kappa
\]  

Where \( V_0 \) is the voltage before inserting the dielectric and \( V \) is the voltage after insertion. Since \( \kappa > 1 \) always, we have

\[
V < V_0
\]  

(5)
Setup

1. Move the Variable Capacitor plates so they are about 2 mm apart. Use the adjustment screws on the back of the moveable plate to make the plates parallel. Easiest way to do this is to look directly down from above the plates and adjust the horizontal adjust until the gap looks uniform, then look at the gap from the side and even with the center of the plates and adjust the vertical screw. May need to repeat the process a few times.

2. Position the movable plate so the leading edge of the indicator foot (see Fig. 3) is at the 0.2 cm position. The gap between the two plates should be 0.2 mm all the way around. Check it with a ruler. If the gap varies repeat step 1. If the gap is not 0.2 mm, release the holding screw on the non-moving plate and move it until the gap is 0.2 mm and then tighten the screw back down.

3. Attach the twin lead (red & black) connector to the Signal Input jack on the Basic Electrometer. Route the wires as far away from where your hand and your body will be as possible. The charges in this experiment all small so static discharge will foul things up. Also, people are conducting plates and have a significant amount of capacitance. You can foul things up just by being close. It is best to make the fixed plate ground by attaching the black wire’s spade lug to it. Attach the red spade lug to the terminal on the moving plate. The wire must be free to move when the plate moves.

4. If you have a black banana/banana wire (not included) attach it as shown from the common (com) terminal on the Electrostatic Voltage Source to the ground terminal on the Electrometer. Alternately, use the provided banana/spade wire and connect the spade lead to the terminal on the fixed plate where the other ground lead is already attached. Attach the red banana/spade lead to the +30V terminal and leave the spade end free. Plug in the transformer and apply power to the Electrostatic Voltage Source. Shift the switch on the back to the On position. The green Power On light should glow.

5. Use the supplied adaptor cable to attach from the Signal Output on the Electrometer to the A Analog Input on the 850 Universal Interface. It is important that it be the A input!
6. In PASCO Capstone, create a table and create a user-entered data set called Separation with units of cm. Enter the values shown in Table I. Select the Voltage measurement in the second column.

<table>
<thead>
<tr>
<th>Separation (cm)</th>
<th>Voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.0</td>
<td></td>
</tr>
<tr>
<td>7.0</td>
<td></td>
</tr>
<tr>
<td>6.0</td>
<td></td>
</tr>
<tr>
<td>5.0</td>
<td></td>
</tr>
<tr>
<td>4.0</td>
<td></td>
</tr>
<tr>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td></td>
</tr>
</tbody>
</table>

7. Create a graph of Voltage vs. Separation.

Procedure A: The Effect of the Plate Separation

1. Set the capacitor plates 0.3 cm apart by setting the movable plate so leading edge of its indicator foot is at the 0.3 cm mark.

2. Turn on the electrometer and set the range button to the 100 V scale.

3. Remove any charge from the capacitor by momentarily touching both plates at the same time with your hand.

4. Zero the electrometer by pressing the ‘ZERO’ button until the needle goes to zero.

5. Momentarily connect a cable from the +30V outlet in the voltage source to the stud on the back of the movable capacitor plate. This will charge the capacitor. Remove the charging cable.

6. Read the following steps. They need to be performed quickly since the charge will slowly escape from the electrometer, especially if the humidity is high. One person should run the computer while one moves the capacitor plate. Everyone else should stay back. Everyone should try to be in the same position for each reading. Anybody who is close is a significant part of the system and can make the readings change.
7. Slide the movable plate so it is at 8.0 cm (leading edge of the indicator foot). Once the plate is in position, the person moving the plate should move away 50 cm or so and try to be in the same position for each measurement.

8. In Capstone, click the PREVIEW button at the lower left to begin collecting data. Colored numbers will appear in first row of the table. The person doing the computer should click the Keep Sample (red checkmark in the lower left) button. The number in the first row will turn black and the colored number will move to the second row. The person at the computer should read the next separation (7 cm) out loud and wait.

9. Move the plate to 7.0 cm and repeat the process until 0.3 cm.

10. Click the STOP button to end the data collection.

11. Examine the graph. If it looks like a smooth curve, you are done. If not, repeat the process until you get a nice looking run.

Analysis A

\[ V = \frac{Q}{\kappa \varepsilon_0 A/d + C_{sys}} \]  

Equation 3

Examination of Equation 3 from Theory A show that if \( C_{sys} = 0 \), then \( V \) is directly proportional to \( d \) and the Voltage vs. Separation graph on the Data page should be a straight line. This is clearly not the case. To verify Equation 3 for the case where \( C_{sys} \) is not zero, we need to know \( Q \) and \( C_{sys} \). We determine these by fitting the math model (Equation 3) to the data.

First we note that

\[ \kappa \varepsilon_0 A = (1.00) \times (8.85 \times 10^{-12} \text{ F/m})( 2.46 \times 10^{-2} \text{ m}^2) = 2.18 \times 10^{-13} \text{ F/m} = 2.18 \times 10^{-11} \text{ F cm}. \]

So the parallel plate capacitance when \( d = 1 \text{ cm} \) is \( C_{1.0} = 2.18 \times 10^{-11} \text{ F} \). Note that this value is entered in line 2 of the Calculator.

When \( d \) is small (0.3 cm) the first term in the denominator dominates and

\[ Q \sim V_{0.3} \times (\kappa \varepsilon_0 A)/d = (30 \text{ V})*(2.18 \times 10^{-11} \text{ F cm})/(0.3 \text{ cm}) = 2.2 \times 10^{-9} \text{ C}. \]

This value is entered as an initial guess for the value of \( Q \) in line 1 of the calculator. \( Q \) is constant so when \( d \) becomes large, \( C_{sys} \) dominates in the denominator and we have:

\[ C_{sys} \sim Q/V_8 \sim 2.2 \times 10^{-9} \text{ C}/80 \text{ V} = 2.7 \times 10^{-11} \text{ F} \]

Where \( V_8 \) is the voltage when \( d = 8 \text{ cm} \). This is taken as the initial guess for \( C_{sys} (=C_1) \) on line 3 of the calculator.
Note that $C_{sys}$ is about equal to $C_{1.0}$ at 1.0 cm. At 0.3 cm, $C_{0.3} = 7 \times 10^{-11}$ F so $C_{0.3} \sim 3 \, C_{sys}$ and the approximation above is decent but not great. At 8 cm $C_8 = 2.7 \times 10^{-12}$ F = $C_{sys}/10$, so the approximation is good, but not perfect.

1. In the Calculator, create the following calculations:

   $Q = 3.0 \times 10^{-9}$ Units of C
   $\kappa \varepsilon_0 A = 2.18 \times 10^{-11}$ Units of (F cm)
   $C_1 = 3.6 \times 10^{-11}$ Units of F
   $V_{\text{model}} = \frac{Q}{(\kappa \varepsilon_0 A/[\text{Separation}]+C_1)}$ Units of V

2. Use the Data Display button ( ) to select your best run.

3. Adjust the values for $Q$ on line 1 of the Calculator and for $C_1$ on line 2 to make the model match the experimental curve as well as possible.

4. Answer the first four questions on the conclusions page.

Procedure B: The Effect of a Dielectric between the Plates

1. In PASCO Capstone, create a table and create a user-entered data set called Paper Position with no units. Enter the values shown in Table II. Select the Voltage measurement in the second column.

   Table II: Paper Dielectric

<table>
<thead>
<tr>
<th>Paper Position</th>
<th>Voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 out</td>
<td></td>
</tr>
<tr>
<td>2 in</td>
<td></td>
</tr>
<tr>
<td>3 out</td>
<td></td>
</tr>
<tr>
<td>4 in</td>
<td></td>
</tr>
<tr>
<td>5 out</td>
<td></td>
</tr>
<tr>
<td>6 in</td>
<td></td>
</tr>
<tr>
<td>7 out</td>
<td></td>
</tr>
<tr>
<td>8 in</td>
<td></td>
</tr>
<tr>
<td>9 out</td>
<td></td>
</tr>
</tbody>
</table>

2. You will use paper as the dielectric to be inserted between the plates. Get a stack of paper about 1 cm thick.

3. Position the movable plate of the capacitor at 8 cm.

4. Turn on the electrometer and set the range button to the 100 V scale.

5. Remove any charge from the capacitor by momentarily touching both plates at the same time with your hand.
6. Zero the electrometer by pressing the ‘ZERO’ button. The needle must be at zero.

7. Momentarily connect a cable from the +30V outlet in the voltage source to the stud on the back of the movable capacitor plate. This will charge the capacitor. Remove the charging cable.

8. Click on the PREVIEW button.

9. One student holds the stack of paper directly above the gap between the capacitor plates so that the long side of the paper is vertical. Hold the paper with one hand and keep the other hand on the metal connector attached to the signal input of the Electrometer so that there is no static charge on the student holding the paper. Press the Keep Sample button to record the voltage when the paper is not between the plates.

10. Lower the paper between the two plates until it touches the base. Do not let the paper touch either plate! Keep your hand as far above the plates as possible. Press the Keep Sample button to record the voltage when the paper is between the plates.

11. Pull the paper back above the plates and repeat steps 8 and 9 several times.

12. Click the STOP button to stop monitoring the data.

13. If the final voltage with the paper out is much different from the initial paper out value, you probably touched the plates and should repeat the experiment.

Conclusions

1. What happened to the voltage as the plates got closer together ($d$ decreasing)?

2. What were your best fit values for the charge $Q$ and $C_{sys}$?

3. How well did your model fit the data? Try to explain any discrepancy. Hint: What approximations are made when deriving the parallel plate capacitance ($C = \kappa \varepsilon_0 \frac{A}{d}$) from Gauss’ Law?

4. Briefly discuss the value of computer modeling.

5. Examine Table II. Does the data agree with Equation 5? What does a dielectric do?