Kirchhoff's Circuit Laws

Equipment

1	Resistive/Capacitive/Inductive Network	UI-5210
1	AC/DC Electronics Laboratory	EM-8656
3	Voltage Sensors	UI-5100
2	Current Probe	PS-2184
1	Short Patch Cords (set of 8)	SE-7123
Required but not included:		
1	850 Universal Interface	UI-5000
1	PASCO Capstone	

Introduction

Kirchhoff's Laws form the basis of all circuit analysis. Here we verify the laws for a resistive circuit using a DC input and for a time varying RC circuit. The DC portion and the RC portion of the lab are each stand-alone labs.

Theory

Kirchhoff's Rules (sometimes called laws) state:

1. Junction Rule: the total current flowing into any point is zero at all times where we use the convention that current into a point is positive and current out of the point is negative.

$$\Sigma I = 0 \qquad (1)$$

2. Loop Rule: the sum of the voltage drops around any closed loop must equal zero where the drop is negative if the voltage decreases and positive if the voltage increases in the direction that one goes around the loop.

$$\Sigma V = 0 \qquad (2)$$

We will apply Kirchhoff's rules to the "Y" circuit shown in Figure 1. Vo will be the supply voltage from Output 1 of the 850 Universal Interface. Vc will be produced by the voltage divider setup shown in Figure 2. We will not attempt to calculate Vc, but will measure it directly. To solve for the three unknown currents, we need three equations. Applying Equation 1 at point A in the circuit:

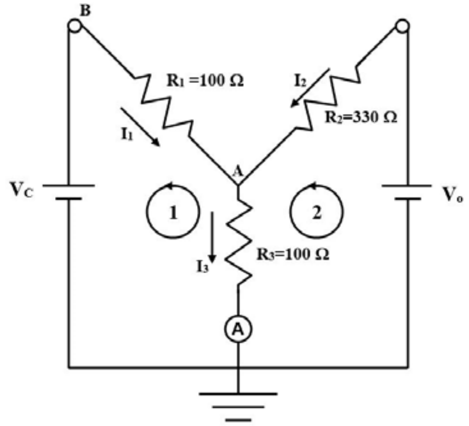
$$I_1 + I_2 - I_3 = 0 \qquad (3)$$

where we take current flowing in as positive and current out as negative. Applying Equation 2 around loops 1 and 2 yields:

$$R_1I_1 + R_3I_3 - V_C = 0 \qquad (4)$$

$$-R_2I_2 - R_3I_3 + V_o = 0 \qquad (5)$$

where the voltage is negative if we go from high voltage to low (with the current arrow across a resistor).



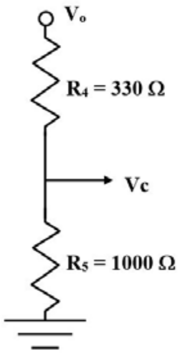


Figure 1: "Y" Circuit

Figure 2: Voltage Divider

Resistor Calibration

1. In PASCO Capstone, open the Hardware Setup and click on Signal Generator #1 on the 850 Universal Interface and select the Output Voltage Current sensor.
2. Plug in a Voltage sensor to Channel A on the interface.
3. Open the Data Summary and click on the Properties gear button for the Voltage Sensor Ch. A. Set the Gain to 1000x.
4. If the Current Probe is not visible in the Data Summary under the Voltage Sensor (Ch. A), click on the eye (visibility) icon next to the Voltage Sensor (Ch. A) and select the Current Probe.
5. Create a graph of Output Voltage vs. Current Probe (Ch. A). Create another graph of Output Voltage vs. Output Current.
6. On the Sampling Control Bar at the bottom of the Capstone page, set the Common Sample Rate to 50 Hz.
7. Construct the circuit shown in Figures 3 through 5. A l00 Ω (± 5%) (brown-black-brown-gold) resistor is connected in series with a Current Probe (the A with a circle around it [for ammeter] on the circuit diagram). A Voltage sensor is attached to the Current Probe as shown and then to Ch. A on the 850 Universal Interface. It is important to observe polarity by connecting red to red and black to black where possible. There is a second ammeter built into the 850 Output 1.

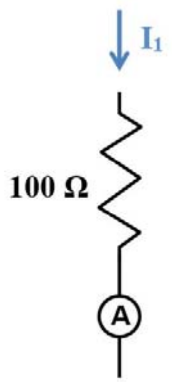


Fig. 3: Calibration Circuit

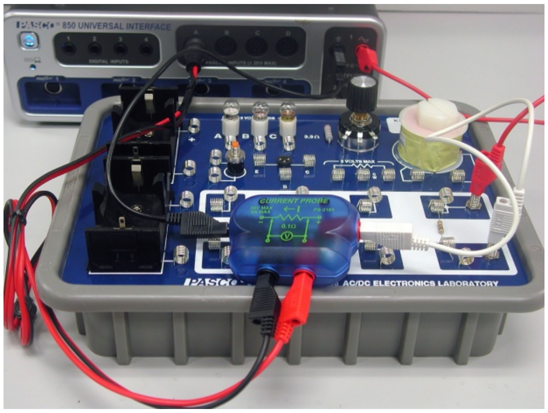


Figure 4: Ammeter Calibration

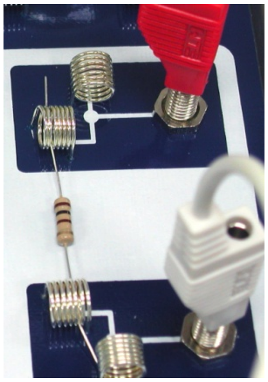


Figure 5: 100 Ω Resistor

The color codes on the resistors only have a precision of ±5%. This can be improved to about ±1% using the calibration circuit.

1. Click open the Signal Generator at the left of the page. Set Waveform to Triangle at 10 Hz and an amplitude of 10 V. Click Auto. Click Signal Generator to close the panel.
2. Click Record. After about 2 seconds, click Stop.
3. For each graph, select a Linear fit.
4. The slope of each line is the measured resistance. Record the slopes and average the two values.
5. Repeat for another 100 Ω and a 330 Ω (orange-orange-brown-gold) resistor. Make sure you keep track of which 100 Ω resistor is R_1 and which is R_3.

Ammeter Calibration

The Current Probes work by measuring the voltage drop across a small resistor (0.1 Ω for the Current Probes). Since the sensitivity is about 0.1 mA, this means the 850 Universal Interface must measure voltages of 0.01 mV. Noise can result in significant zero error. By averaging over several seconds we can achieve a precision of 0.1-0.2 mA, but with systematic errors that can approach 1 milliamp. We can correct for this with a brief calibration procedure.

1. Make sure the 100 Ω resistor is back in the calibration circuit (see Figure 4).
2. Click open the Signal Generator at the left of the screen. Set 850 Output 1 for a DC Waveform and a DC Voltage of 0 V. Click the On button.
3. Create a Digits display with Current Probe (Ch.A). Select units of mA and in the Statistics, turn on the Mean.
4. Click Record.

5. Wait several seconds until the measured mean current stops varying as the average becomes well defined. Click Stop.
6. Create a table as shown below. In the first column, create a User-Entered Data Set called "Theory Current" with units of mA. In the second column, create a User-Entered Data Set called "A Current" with units of mA. In the third column, create a calculation called "A Correct" with units of mA:

   A correct = -[A Current (mA)]+[Theory Current (mA)]

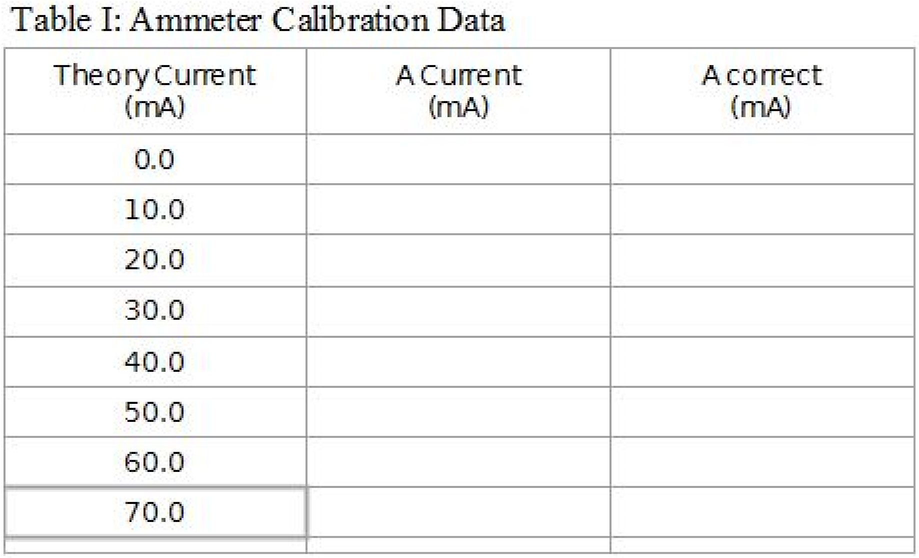


Table I: Ammeter Calibration Data

Theory Current (mA)	A Current (mA)	A correct (mA)
0.0		
10.0		
20.0		
30.0		
40.0		
50.0		
60.0		
70.0		

7. Enter the value in the second column of the first row of Table I.
8. Click Delete Last Run at the bottom of the screen.
9. In the Signal Generator panel, increase the voltage by 1 V and repeat. Then repeat, increasing the voltage by 1 V each time until 7 V is reached. Enter the values in the appropriate line. Turn Signal Generator off.
10. The Theory Current in column 1 is calculated assuming that the resistance is exactly 100 Ω. If your value is different, correct the first column by calculating I = V/R using your exact R.

DC Setup

Construct the circuit shown in the Figure 6:

1. First attach the resistors as shown in Figure 7. Note that the "Y" is on the left and the voltage divider (see Figure 2 on the Theory page) is the two resistors on the right. R_1 is the 100 Ω resistor on the upper left.
2. R_2 is the 330 Ω resistor at top center.
3. R_3 is the 100 Ω resistor running from top to bottom.
4. The voltage divider will supply Vc to point B on the left end of R_1. Attach a 330 W resistor from top to bottom as shown and a 1 kΩ resistor (brown-black-red-gold) at the

bottom right. A white jumper wire connects the upper left (left end of R_1) to the lower right (center of the voltage divider providing Vc). A white jumper wire also connects the right end of R_2 to the upper right spring where Vo will be applied. Another jumper connects the lower left spring to the left end of the 1000 Ω resistor.

5. Three more jumper wires have one end attached to springs as shown. *Note: it is important to use the jumper wires to attach to the board and not connect to the springs directly with alligator clips since it is rather easy to pull the springs loose from the circuit board.*

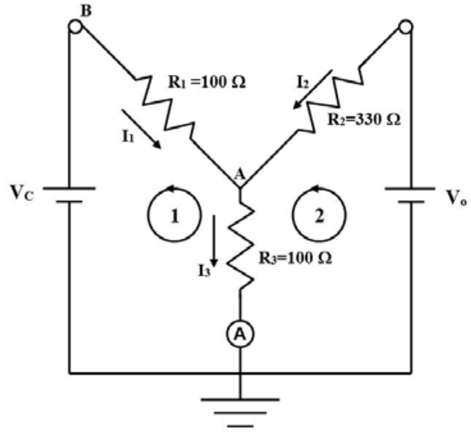


Figure 6: "Y" Circuit

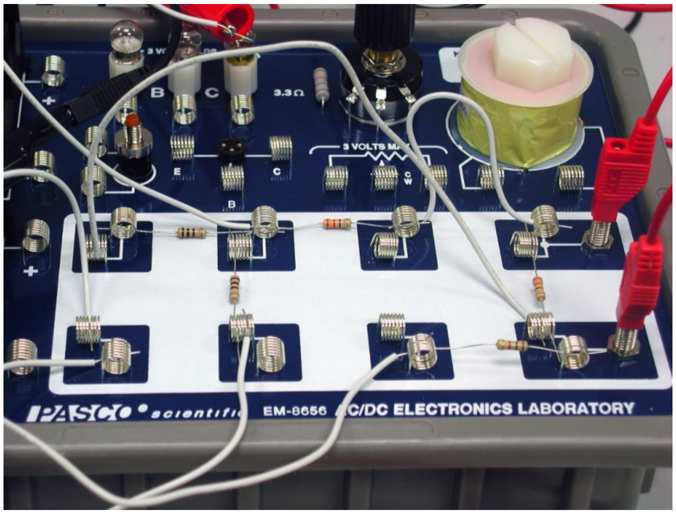


Figure 7: Resistor Setup

6. Attach the ammeter by clipping the red end of the ammeter to the white wire attached to the bottom of the 100 Ω resistor (R_3) as shown in Figure 8. The black side of the ammeter attaches to the white wire coming from the lower left spring. The black wire from Output 1 on the 850 also attaches to this point (ground).
7. Plug Voltage sensors into Channels B and C on the 850. The black leads from both are attached to the black side of Output 1 on the 850. The red side of Output 1 is attached to the upper banana input (Vo) on the circuit board as shown in Figure 9. The red lead from Analog input C is attached to the lower banana input (Vc) on the circuit board. The red lead from Analog input B is attached to the white wire coming from the junction point A between the three resistors. It will measure Va.

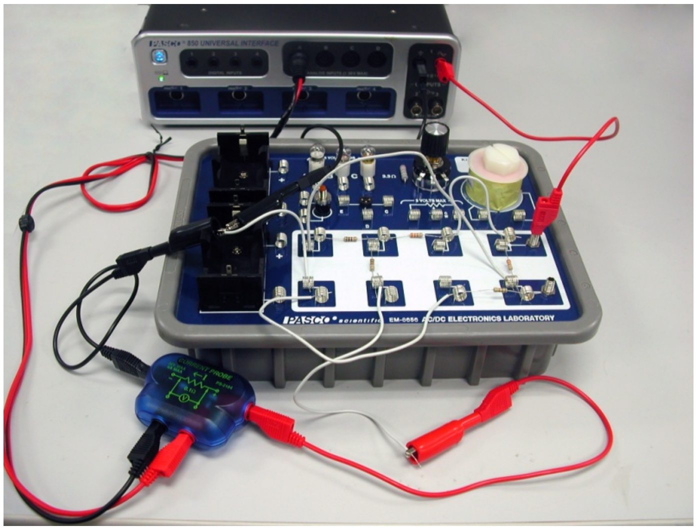


Figure 8: Hooking Up the Ammeter

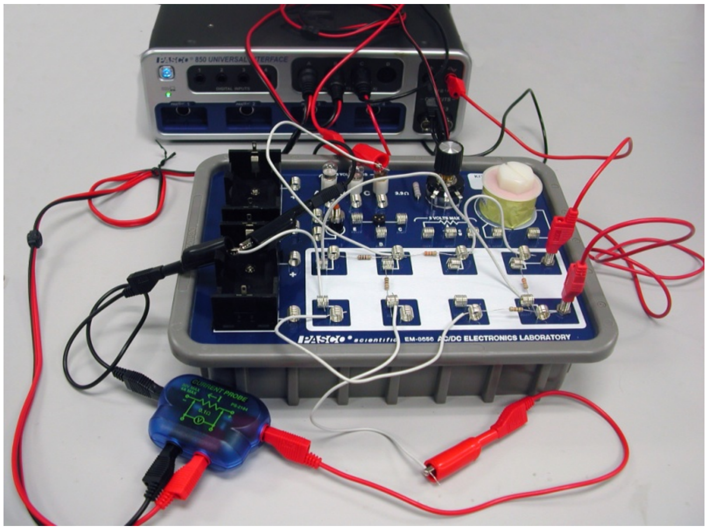


Figure 9: Connecting the Voltage Probes

DC Current Procedure

1. Click open the Signal Generator at the left of the screen. Set 850 Output 1 for a DC Waveform and a DC Voltage of 15 V. Click the Auto button.
2. Open the Calculator and create two calculations, both with units of A:

   I1 cal = ([Output Voltage, Ch O1 (V)]-[Voltage, Ch B (V)])/[R2-330 (Ω)]

   I2 cal = ([Voltage, Ch C (V)]-[Voltage, Ch B (V)])/[R1-100 (Ω)]
3. Create Digits displays for V(Ch. B), V(Ch. C), Output Voltage, Current Probe (Ch. A), I1 cal, and I2 cal. For the Current Probe Digits, choose the Mean on the statistics.
4. Click Record.

5. Wait several seconds until the measured current stops varying as the average from the Current Probe becomes well defined. Click Stop.
6. Record the voltage and current values from all the Digits displays.

Note: Although the total current, I3, is measured directly, I1 and I2 are calculated from the measured voltage drops using ΔV/R = I. Although it would be nice to measure them directly, shifting the ammeter ground introduces error of a few mA which decreases the precision of the experiment since the current values are quite small. This really is not a limitation on the results since this is what the ammeter does as well. It measures the voltage drop across a 0.1 Ω precision resistor. We have just made our own ammeters by replacing the 0.1 Ω resistor with the known values for the resistors in the circuit.

DC Analysis

1. Apply the correction from the Current Correction graph to i_A ave to get a corrected value for I3 and record it.
2. Using Equations 3-5 from Theory, calculate values for I_1, I_2, and I_3. These three equations may be solved simultaneously, or you may solve them using a 3x4 augmented matrix.
3. Calculate the theory values that you should expect for the three currents and the percent difference from measured and record them.

Conclusion

How well did your values check out?

RC Setup

Construct the circuit shown in the Circuit Diagram (Figure 10) with reference to Figures 11 and 12:

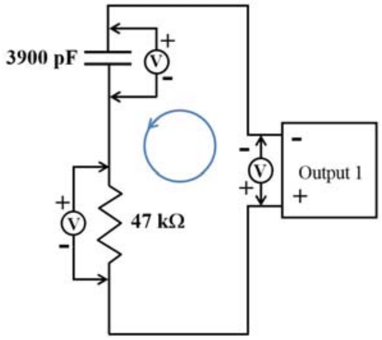


Fig. 10: Circuit Diagram

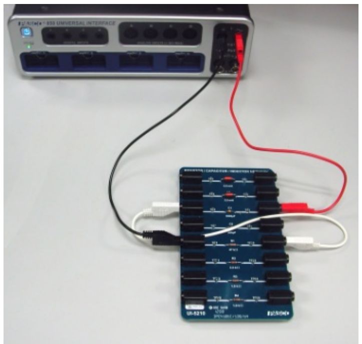


Figure 11: RC Series

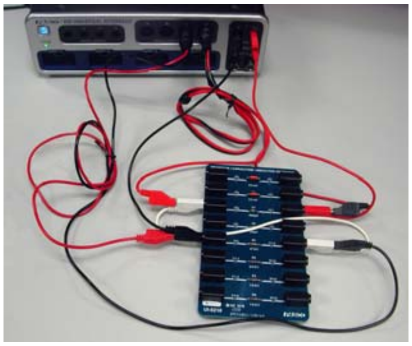


Figure 12: Adding the Sensors

1. First construct the series circuit shown in Figure 11 using the 3900 pF capacitor and the 47 kΩ resistor. Note the polarities, with the red lead from the 850 Output 1 attached to

the right side of the capacitor and the left side of the capacitor attached to the right side of the resistor.

2. Now add the Voltage Sensors as shown in Figure 12. The polarities must match that shown in Circuit Diagram (Fig. 10) with the red leads on the left ends of the resistor and capacitor. The Voltage Sensor across the resistor must attach to Channel C and the Voltage Sensor across the Capacitor must attach to Channel D.
3. Create an Oscilloscope display with both the voltages from Channel C and D on the same vertical axis and time on the horizontal axis.
4. Set the sample rate for Voltage Ch. C and Ch. D at 500 kHz. Change Sampling Mode to Fast Monitor Mode.

RC Procedure

1. Click open the Signal Generator. It should be set for a Square Waveform at 1000 Hz and 10 V. Click On.
2. Click Monitor at the lower left of the screen. The oscilloscope should record one cycle and stop. If any of the vertical jumps in the square wave fall exactly on one of the vertical time lines (0.0002 s, etc.), click monitor again.
3. Click Off on the Signal Generator and click on the Signal Generator button to close the Signal Generator panel.
4. The pattern on the oscilloscope screen shows the input voltage (V0), the voltage across the resistor (VR), and the voltage across the capacitor (VC). Why the voltages vary will be dealt with later in the course. Here we are only interested in verifying that Kirchhoff's Loop rule holds at any instant of time.

RC Analysis

1. Create a table as shown below.

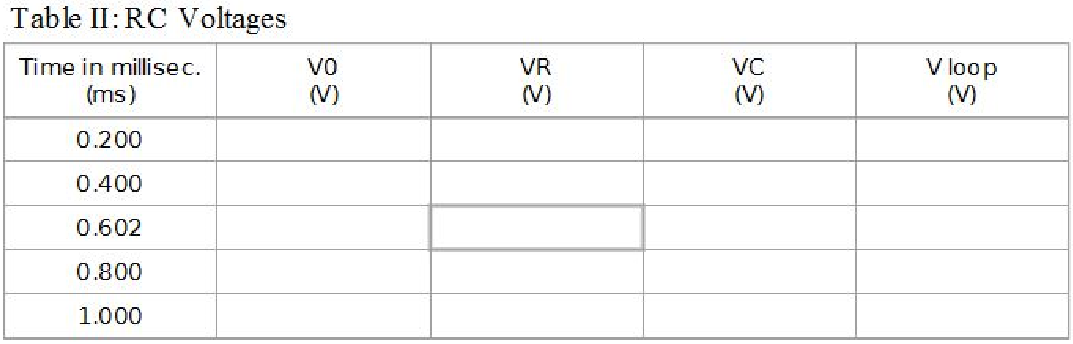


Table II: RC Voltages

Time in millisec. (ms)	V0 (V)	VR (V)	VC (V)	V loop (V)
0.200				
0.400				
0.602				
0.800				
1.000				

Create the User-Entered Data sets called "Time in millisec" with units of ms; "V0" with units of V; "VR" with units of V; and VC with units of V. In the last column, create a calculation with units of V:

V loop = [V0 (V)]+[VR (V)]+[VC (V)]

2. Click on the Coordinate tool on the oscilloscope toolbar (crosshairs). We want 5 significant figures in the coordinates box. If that is not the case, right click on the center of the cross-hairs on the oscilloscope and select Tool Properties and increase the significant figures to 5.
3. Move the cross-hairs up the 0.2 ms (0.0002 s) line and record the values for V0, VR, and VC to three decimal places in the Table II. Try to get the value for time in the coordinates box as close to 2.00×10^{-4} s as possible (the last two digits will always be zero). You should be able to get within 0.02×10^{-4} s always and generally exactly on 2.00×10^{-4} s. If you can't get it exactly on, try to take all three data points at the same time.
4. Repeat for 0.4 ms, 0.6 ms, 0.8 ms, and 1.0 ms.

RC Conclusions

Considering the sum voltages in the 5th column of the RC Voltages table under the RC Analysis tab, what can you conclude about Kirchhoff's Loop Rule for time varying circuits?