Physical Pendulum Minimum Period

Equipment

1	Rotary Motion Sensor	PS-2120A
1	Rotational Accessory	CI-6691
1	Large Rod Base	ME-8735
1	45 cm Rod	ME-8736
	Required but not included:	
1	Balance	SE-8723
1	Meter Stick	SE-8827

Introduction

The period of the physical pendulum shown in Figure 1 is measured with the lower brass mass in various positions. If you start with the mass at the bottom, and then move it up to the position shown, you find that the period has decreased, due to the fact that you have decreased the pendulum's rotational inertia. However, if you move the mass up close to the pulley, the period will be very large, due to the fact the pendulum is almost balanced by the upper fixed mass. Somewhere in between these two extremes, the period goes through a minimum value.

Setup

1. Use the base and 45 cm rod to support the Rotary Motion Sensor as shown in Figure 1. Plug in the sensor.

2. In PASCO Capstone, set the sample rate to 50 Hz. Create a graph of Angle (in degrees) vs. Time.

3. Measure the mass of the black rod, M_{rod}, and record below.

4. Measure the mass of each of the two brass masses. They should be about the same: If they are slightly different, use an average value for "m".

5. Measure the length of the black rod, L.

Figure 1: Minimum Period of a Pendulum

Theory

The general formula for the period of a physical pendulum is

$$T = 2\pi\sqrt{I/mgx} \qquad (1)$$

For the pendulum shown here, the inertia, I, is the total inertia of the rod plus brass masses. The upper mass is fixed at r_0 and the lower mass is moved to various positions, r. Both have the same mass, m. Thus

$$I = I_{rod} + mr_o^2 + mr^2 \qquad (2)$$

A thin rod of mass, M_{rod}, and total length, L, rotating about center of mass, has rotational inertia

$$I_{rod} = (1/12)\, M_{rod}\, L^2 \qquad (3)$$

The denominator in Equation (1) is the generic form of the restoring torque. The rod is balance at its center, so only the two masses create a torque. Note that they create opposite torques, and thus

$$\text{"mgx"} = \text{Torque} = mgr - mgr_o \qquad (4)$$

Substituting Equation (2) and Equation (4) into Equation (1) yields

$$T = 2\pi\sqrt{\frac{I_{rod}+m\left(r^2+r_o^2\right)}{mg(r-r_o)}} \qquad (5)$$

To find the position, r, where the period from Equation (5) is at a minimum, take the derivative with respect to "r", and set that expression equal to zero. Setting dT/dr = 0 and simplifying yields

$$mr^2 - 2mr_or - (I_{rod} + r_o^2) = 0 \qquad (6)$$

Finally, using the quadratic equation for "r" in Equation (6), and substituting that $\Delta r = r - r_o$, yields the position for minimum period is

$$\Delta r_{min} = \sqrt{2r_o^2 + I_{rod}/m} \qquad (7)$$

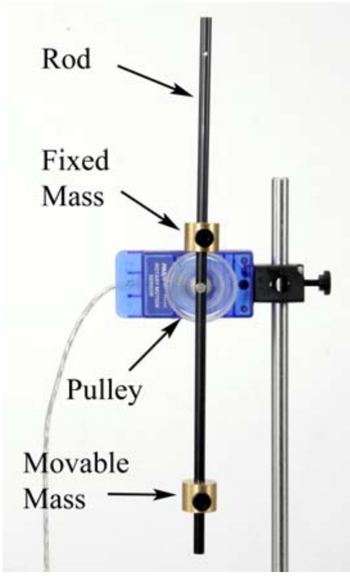

Figure 2: Physical Pendulum

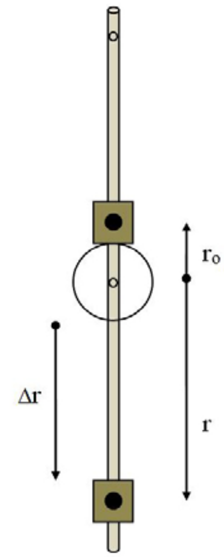

Figure 3: Mass Positions

Calculate Minimum Period

1. Place the brass masses on the black rod as shown in Figure 2. The upper mass should be up against the clear pulley, and is never changed during the lab.

2. Measure the distance r_o as shown in Figure 3. This is the distance from the pivot point up to the center of the fixed mass.

3. Calculate the rotational inertia of the black rod using Equation (3). What are the units?

4. Use Equation (7) to calculate the theoretical position, Δr, of the movable mass for when the period is at a minimum.

5. Place the movable mass at the end of the rod, and use the meter stick, as shown in Figure 4, to adjust the length, Δr, to 15 cm. Remember that $\Delta r = r - r_o$ is the distance from the edge of the pulley out to the near edge of the mass. See Figure 3.

6. In the next section, as you move the lower mass to new positions, always measure the distance in this same manner.

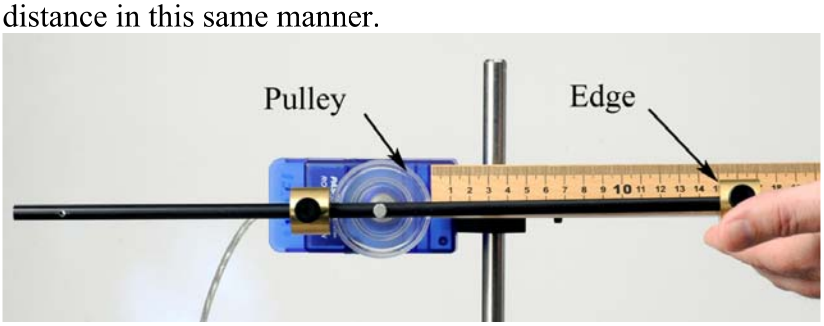

Figure 4: Measuring Δr

The position, r, of the mass is from the axis of rotation out to its center of mass, as shown in Figure 3. Since this is difficult to measure exactly, we instead measure Δr, the distance from the edge of the pulley out to the near edge of the mass, as shown here. Since the theoretical position for the minimum period is written for Δr, you never have to deal with the actual distance, r.

Measuring Period

7. Re-measure the position and confirm that Δr is 15 cm.

8. Take a run of data for very small angles. The amplitude **<u>must</u>** be less than 5°. Record 5 to 10 periods.

9. Create a table as shown below. Create a User-Entered Data set called ω with units of rad/s. Create a calculation in Capstone and select it for the second column:

   Period=2*π/[ω (rad/s)]   with units of s

   Table I: Calculate Period

ω (rad/s)	Period (s)

   Turn on the Mean in the Statistics.

10. Use a Damped Sine fit to find the angular frequency, ω, and record the value in Table I. Make sure the curve fit is set for at least four digits.

11. Take several runs. Note that the average value is being calculated, and that the period is calculated also.

Changing Δr

12. Create a table as shown below. Create a User-Entered Data set called Δr with units of cm and another set called T with units of s.

13. Enter the average Period for 15 cm in Table II.

14. Move the mass up so that Δr is 14 cm. Take several runs and find the average period, as before. Don't forget to clear out the previous values in Table I for each new position.

15. Repeat for all the Δr positions listed in Table II.

Table II: Position

Δr (cm)	T (s)
15	
14	
13	
12	
11	
10	
9	
8	
7	
6	
5	
4	

Analysis

16. Create a graph of T vs. Δr. How does your minimum value compare to the predicted?

17. You only took data every even centimeter. You can go back and take a few more runs at 1/2 cm intervals just before and just after the minimum. Insert these new data rows in Table II.

18. Try a Polynomial curve fit on your data. Change the number of parameters to get the best fit.

19. What does the curve fit show for the position of minimum period? Compare this to the predicted value.