Lab 11: Bend Testing Beams

Introduction

A Three-Point Bend Test is performed on plastic beams as shown in Figure 1. As a downward force (F) is applied in the middle of the beam, the flex (Δx) is recorded. The ratio (F/Δx) is the effective stiffness of the length of beam being tested, and is measured directly from the slope of the F vs. Δx graph. The Flexural Elastic Modulus for the material is then calculated.

This experiment uses the ABS plastic beams from the PASCO Structures System. You will need to cut each beam to a length of about 10 cm, so that it will fit between the drive screws.

Equipment

Qty	Items	Part #
1	Materials Testing Machine	ME-8236
1	Bending Accessory	ME-8237
1	Structures Flat Beam	ME-6987
1	Structures Thin I Beam	ME-7012
1	Calipers	SE-8710

Written by Jon Hanks

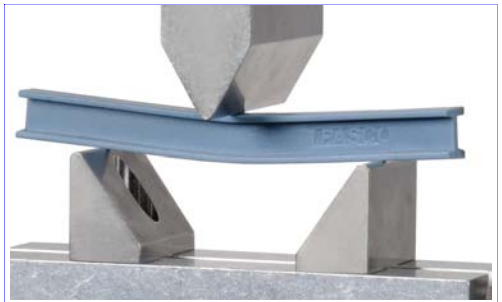


Figure 1. Three-Point Bend Test

Note: You will probably want to make a Compliance Calibration (using the Calibration Rod) before attaching the Bending Accessory! A max force of 500 N is adequate for this experiment.

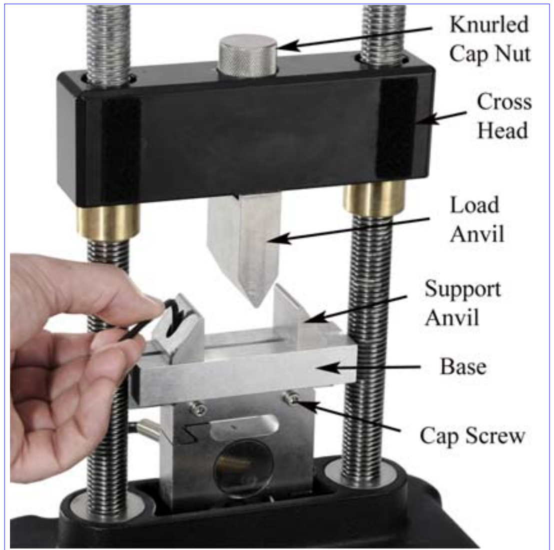


Figure 2. Setting Anvil Spacing

Setup

1. The ME-8237 Bending Accessory consists of two major parts: The upper load anvil and the lower base with the two support anvils. The load anvil sticks up through the cross-head and is held in place by the knurled cap nut.

2. The base (for the support anvils) fastens directly to the load cell using the two cap screws as shown in Figure 2

3. Each anvil is captured by the T-slot in the base, and their separation should always be adjusted so that the Load Anvil is centered between them. Use calipers to make this alignment as accurate as possible. Set the anvil spacing between 5 and 6 cm.

Rectangular Beams

A test sample is supported by two anvils separated by a length, L, as shown in Figure 3. A load, F, is applied in the middle, an equal distance from each anvil, and the resulting flexure (Δx) is measured. The ratio ($F/\Delta x$) is the stiffness of the sample, and depends on the length, L. It also depends on the shape and area of the sample cross-section, as well as the material.

If "E" is the Flexural Elastic Modulus for the material, and "$\mathbf{I}$" is the Area Moment of Inertia for the sample, then

$$F/\Delta x = 48\mathbf{I}E/L^3 \qquad \text{Eqn. (1)}$$

Solving for E, yields

$$E = (F/\Delta x)\, L^3 / 48\mathbf{I} \qquad \text{Eqn. (2)}$$

where the ratio $F/\Delta x$ is determined from the slope of the force vs. flexure graph. The Area Moment of Inertia depends on the cross-sectional shape and area of the sample. For a beam with a rectangular cross-section

$$\mathbf{I}_{rectangle} = (1/12)\, A\, h^2 \qquad \text{Eqn. (3)}$$

where A is the cross-sectional area, and the height, h, is the dimension that is parallel to the applied force. The base, b, is the dimension perpendicular to the applied force, as shown in Figure 4, and since A=bh, Eqn. (3) can be written as

$$\mathbf{I}_{rectangle} = (1/12)\, b\, h^3 \qquad \text{Eqn. (4)}$$

Thus in Figure 5, b>h for the upper picture with the beam being bent in the "weak" direction, and b<h in the lower picture with the beam being bent in the "strong" direction.

Note. The Flexural Modulus is technically not the same as Young's Modulus. Bend testing involves both tensile and compressive stresses, and for some materials these moduli are different.

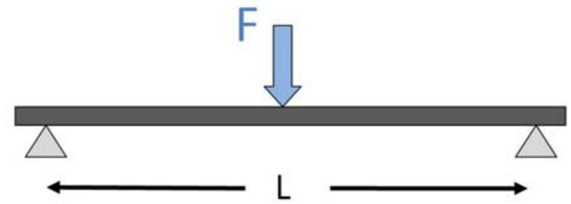


Figure 3. Three-Point Bending Test

Figure 4. Beam with rectangular cross-section

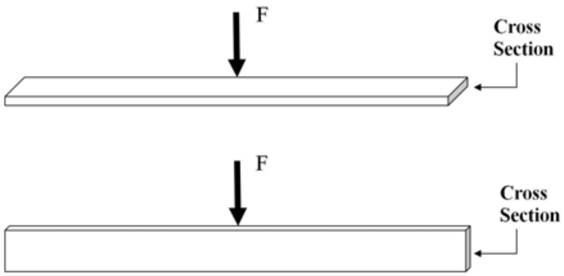


Figure 5. Two ways to bend the beam.

Procedure

1. Carefully measure the length, L, from the top of the camber on each anvil. You can also just measure between the vertical surfaces, and calculate L by including the 1.5 mm radius on each anvil. Record this value below. What is the uncertainty?

2. Cut a 10 cm length of rectangular cross-section beam from the ME-6987 Structures Flat Beam set. You can use either the F4 or 3X4 beams from that set. The shorter 2X3 beam (also in that set) has a smaller cross-section and should be saved for further investigations, later.

3. Measure the cross-sectional dimensions of the beam and record.

4. Use Eqn. (4) to calculate the Area Moment of Inertia, for the beam being bent in both the strong and weak directions. Record these in the table.

5. Place the beam across the support anvils as shown in Figure 6. This will bend the beam in the strong direction. Turn the crank counter-clockwise until the load anvil is just touching the sample.

"Seating" the Test Sample and Setting Pre-Load

Proceed to the next page to test your sample. Your data will look better if you use the normal procedure to "seat" the test sample. If you use a pre-load, do not go over 5 N, as the forces required to bend the beams in this lab are quite low. Remember that you should use the same method for testing your sample, as you used when performing a compliance calibration with the Calibration Rod.

Note: Do not exceed the Yield Strength of the material! If the sample is permanently bent, you went too far. For these samples, you only need to apply a max force of 100 to 200 N.

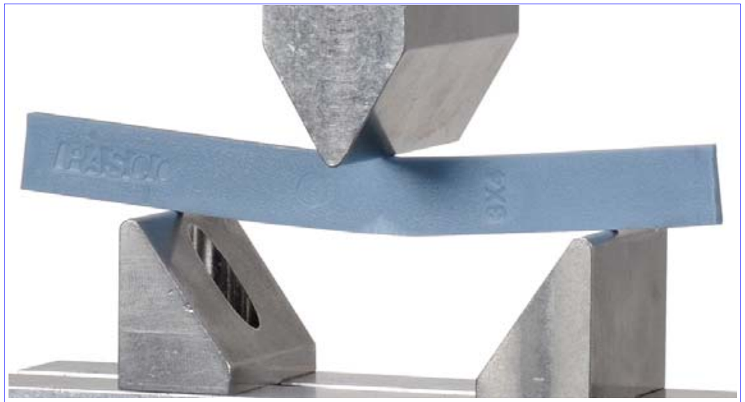


Figure 6. Bending Rectangular Beam

	Rectangle	I (m^4)
1	Strong Direction	2.295E-10
2	Weak Direction	1.510E-11
3		
4		

L= 57.1 mm ± .2 mm
Cross section = 2.61 mm x 10.18 mm
$\mathbf{I} = (1/12)\, b\, h^3$
$=(1/12)\ (.00261\ m)\ \ (.01018\ m\ )^3 = 2.295\ x10^{10}\ m^4$

$=(1/12)\ (.01018\ m)\ \ (.00261\ m\ )^3 = 1.51\ x10^{11}\ m^4$

Taking Data

Note: The sample rate is set to 5 Hz, but you can change this if needed. In general, a slower rate gives smoother (less noisy) data.

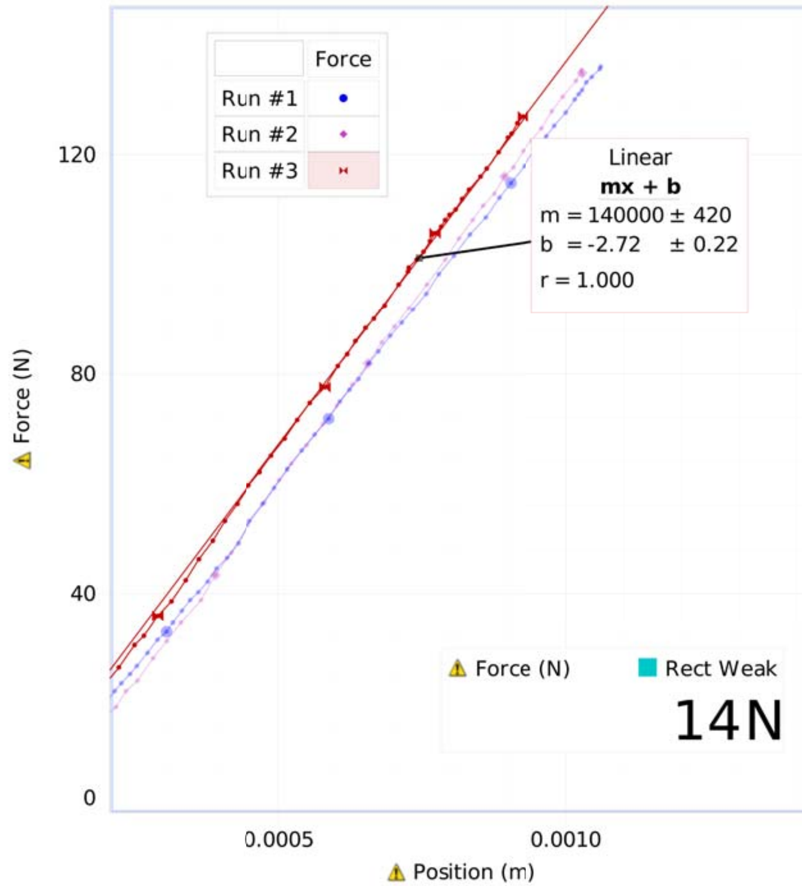
1. Click on Record. Turn the crank counter-clockwise, bending the sample. Increase the force to about 100 N.

2. Click on Stop. The data should be fairly linear. It is OK if there is a slight curvature at the beginning or end, but if there is not a straight section in the middle, you probably have something wrong.

3. To take another run of data, turn the crank clockwise to completely remove the tension.

4. Use a linear curve fit to find the slope. This is the stiffness (F/Δx) of the length of beam you are testing. Take multiple runs and record your values in the table at right to get a good average value. Enter your final average value for F/Δx (slope) in the table below.

	Rectangle	I (m⁴)	Graph Slope (N/m)
1	Strong Direction	2.295E-10	141000
2	Weak Direction	1.510E-11	9200
3			
4			



Three-Point Bending

Average Slope

	Set Slope (N/m)
1	136000
2	143000
3	143000
4	
5	
6	
7	
8	
9	
10	
Mean	140667
Std. Dev.	4041

Note: Enter your slopes in the table above. The average value is calculated for you. Keep taking runs until you get a good solid average for F/Δx, then record only this average value in the table at left.

Analysis

1. Use Eqn. (2) to calculate "E", the Flexural Elastic Modulus. What are the units?

2. Estimate the uncertainty in your value for E.

3. How does the Flexural Modulus compare to the value found in reference data tables for Young's Modulus for ABS plastic?

4. Use Eqn. (1) and your value for the modulus to predict the stiffness of the beam bent in the "weak" direction.

5. Go back to the previous page and test the beam in this configuration. Compare to your predicted value.

6. For this beam, what is the stiffness ratio for strong/weak?

7. In theory, if you shorten the length of the test sample by a factor of 2, what happens to the stiffness of that section?

8. For this case, what would happen to the calculated value for E?

	Rectangle	I (m^4)	Graph Slope (N/m)
1	Strong Direction	2.295E-10	141000
2	Weak Direction	1.510E-11	9200
3			
4			
5			

Answers:

1. $E = (F/\Delta x)\, L^3 / 48\mathbf{I}$
$= (141000)\,(.0571)^3 / 48(2.295\text{x}10^{-10}) = 2.38$ GPa

2. Both the uncertainty in slope and the uncertainty in the measurement of L yields an uncertainty for E of about ± .03 GPa

3. Reference tables give E = 2.3 GPa for ABS. The two moduli seem to be the same.

Easy direction

4. $F/\Delta x = 48\mathbf{I}E/ L^3$
$= 48(1.51\text{x}10^{-11})\,(2.38\text{x}10^{9}\text{ Pa}) / (.0571)^3$
$= 9300$ N/m

5. Measured was 9200 N/m, about 1%

6. ratio = 15/1. 15 times stiffer in the "strong" direction.

7. If shorten length, increase stiffness by factor of 2^3 = 8X $F/\Delta x \propto 1/L^3$

8. E would stay the same, a property of the material, not the shape.

Procedure

1. The **I** Beam can also be bent in two directions, as shown in Figure 7. Measure the cross-sectional dimensions of your beam, and use Eqn. (3) to calculate the Area Moment of Inertia for the beam bending in the weak direction. You can assume that the cross-section is composed of three rectangles.

2. Calculate the Area Moment of Inertia for the beam bending in the "strong" direction. You can once again assume that the cross-section is composed of three rectangles, but you must include an additional term for the flanges. Eqn. (3) gives the Moment of Inertia (I_{cm}) of the area about its own centroidal axis, but you must calculate the moment about the center of the entire beam. Using the parallel axis theorem,

$$I = I_{cm} + Ax^2 \qquad \text{Eqn. (5)}$$

where x is measured from the center of the **I** Beam, to the center of the flange as shown in Figure 7. Record your values for the Area Moment of Inertias in the table, below.

3. Use Eqn. (1) and your value for the modulus to predict the stiffness of the beam bent in both directions. Record in the table, below.

	Beam	Beam Moment (m^4)	F/Δx (N/m)
1	Strong Direction	3.22E-10	197000
2	Weak Direction	4.56E-11	28000
3			
4			

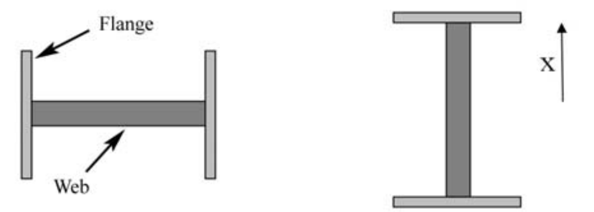


Figure 7. Weak and Strong Directions.

Flange: thickness = 0.97 mm width = 6.42 mm
Web: thickness = 1.61mm width = 8.02mm

I_{weak} = (1/12) (.00802) (.00161)^3 + 2(1/12) (.00097) (.00642)^3
= 4.56x10^{-11} m^4

For strong direction, x=(8.02 + .97)/2 = 4.495 mm

$I_{flange} = I_{cm} + Ax^2$ = (1/12)bh^3 + bhx^2
= (1/12)(.00642)(.00097)^3 + (.00642)(.0097)(.004495)^2
4.88x10^{-13} + 1.258x10^{-10} = 1.263x10^{-10} m^4

I_{web} = (1/12) (.00161) (.00802)^3 = 6.921x10^{-11} m^4

I_{strong} = 2(1.263x10^{-10}) + 6.921x10^{-11} = 3.22x10^{-10} m^4

F/Δx (weak) = 48IE/ L^3
= 48(4.56x10^{-11}) (2.38x10^9 Pa) / (.0571)^3
= 28,000 N/m

F/Δx (strong) = 48(3.22x10^{-10}) (2.38x10^9 Pa) / (.0571)^3
= 197,000 N/m

4. Follow the same procedure as before to record Force vs. Position data for the **I** beam bent in both the strong and weak directions.

5. Use a linear curve fit to find the slope (stiffness) of the beam and compare to the predicted (calculated) values.

6. For this beam, what is the stiffness ratio for strong/weak?

7. Which beam is stronger, your rectangular beam or the **I** beam? Calculate the cross-sectional area for each. What do you conclude?

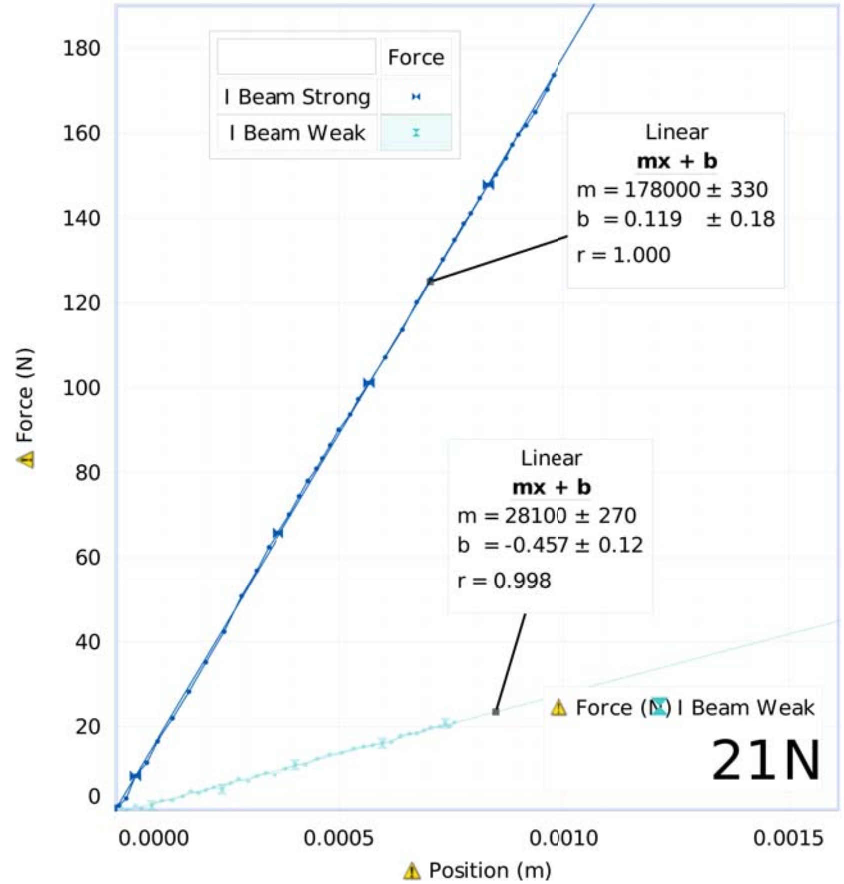
8. For further study, there are other beams you can measure. The ME-7011 polycarbonate beams have the same cross-section as the ME-7012, and the other I-beams in the Structures System are made of ABS but have a bigger cross-section.

Stiffness in Strong direction = 178,000 N/m about 10% off
Stiffness in Weak direction = 28,100 N/m less that 1% off

ratio = 7/1. 7 times stiffer in the "strong" direction.

Both have same area of about 25-26 mm² , but **I** beam is stronger in both directions. Better strength/ weight.

	Beam	Beam Moment (m⁴)	F/Δx (N/m)
1	Strong Direction	3.22E-10	197000
2	Weak Direction	4.56E-11	28000
3			
4			



Three-Point Bending