

## Lab 09: Three-Point Bending

### Introduction

A Three-Point Bend test is performed on a round rod as shown in Figure 1. As a downward force ( $F$ ) is applied in the middle of the rod, the flex ( $\Delta x$ ) is recorded. The ratio ( $F/\Delta x$ ) is the effective stiffness of the length of rod being tested.

The distance between the anvils (see inset) is varied, and the resulting effect on the stiffness of the beam is measured. A graph of the resulting data yields the Flexural Elastic Modulus for the material.

Qty	Items	Part #
1	Materials Testing Apparatus	ME-8236
1	Bending Accessory	ME-8237
1	Shear Sample (Steel)	ME-8240
1	Calipers	SE-8710

*Written by Jon Hanks*

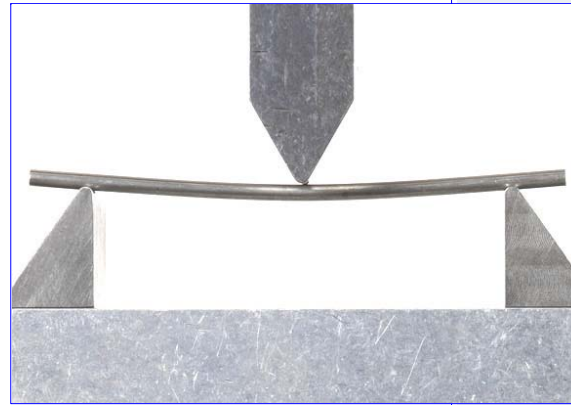
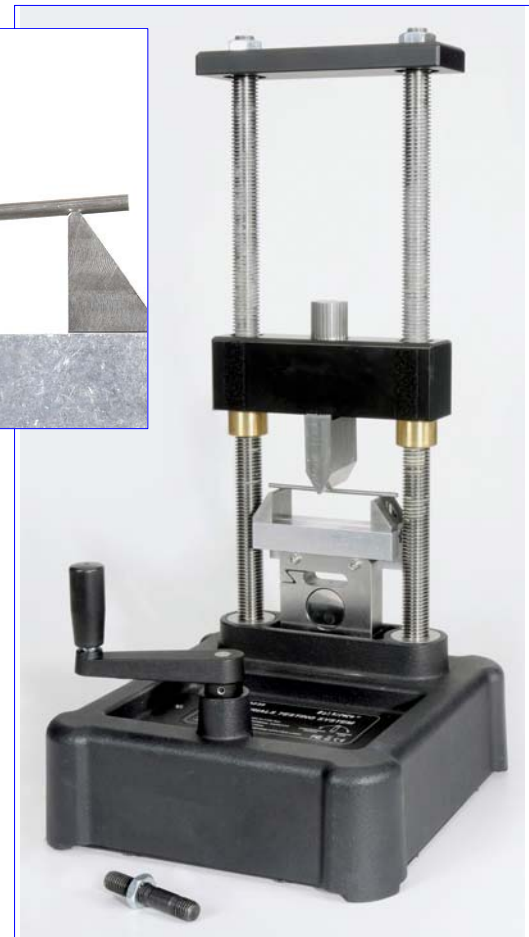


Figure 1. Three-Point Bending

This experiment uses the round rod from the ME-8240 Shear Samples. The lab is written for the steel samples, but any could be used, including samples of your own.

You will need to cut the rod to a length of about 10 cm, so that it will fit between the drive screws.



### Installing Bending Accessory

Note: The ME-8237 Bending Accessory consists of two major parts: The upper load anvil and the lower base with the two support anvils. In this lab, you will vary the support anvil spacing using the hex wrench as shown in Figure 2. Start with the anvils spaced as far apart as possible: If you want, you can reverse the anvils to increase this maximum length.

1. The load anvil sticks up through the cross-head and is held in place by the knurled cap nut, as shown in Figure 2.
2. The base (for the support anvils) fastens directly to the load cell using the two cap screws. Each anvil is captured by the T-slot in the base, and their separation should always be adjusted so that the Load Anvil is centered between them. Use calipers to make this alignment as accurate as possible.
3. Start with the anvils spaced as far as possible, and measure their separation. This length,  $L$ , is to the top of the camber on each anvil. You can also just measure between the vertical surfaces, and calculate  $L$  by including the 1.5 mm radius on each anvil.

### Compliance Calibration

Note: Before you can test the sample, you must complete the steps on the next page. The procedure is written for using a test sample (see Fig. 1), but you should also use this method when performing a compliance calibration of the Materials Tester using the Calibration Rod. In this procedure, you load and unload the system several times to remove all the slack and properly "seat" the test sample, in addition to introducing a small pre-load to the system.

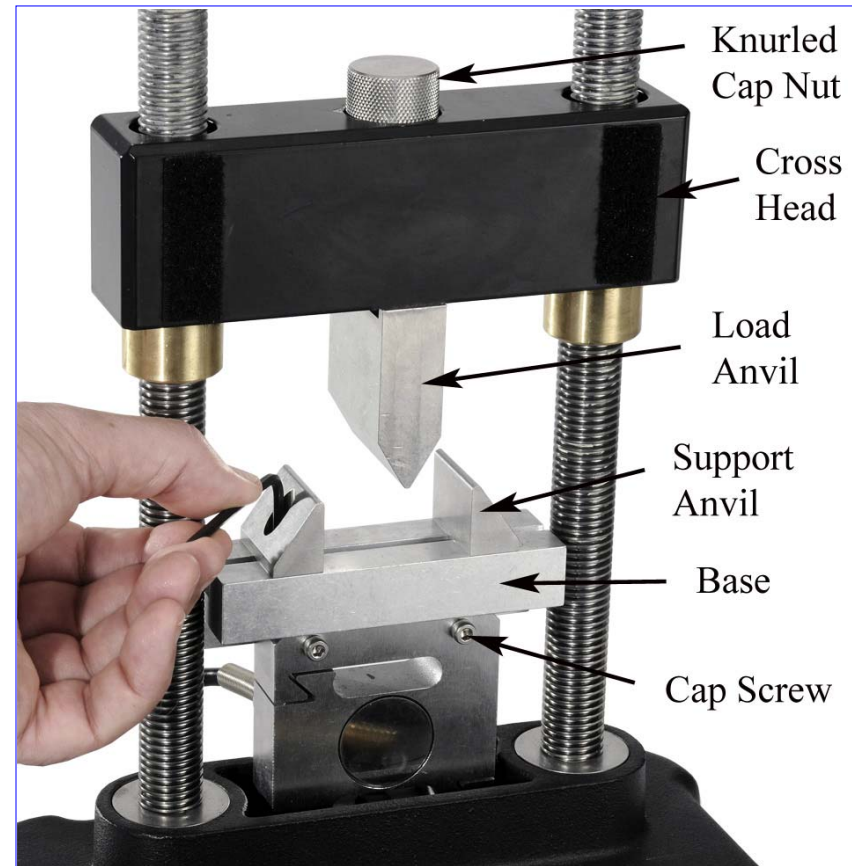


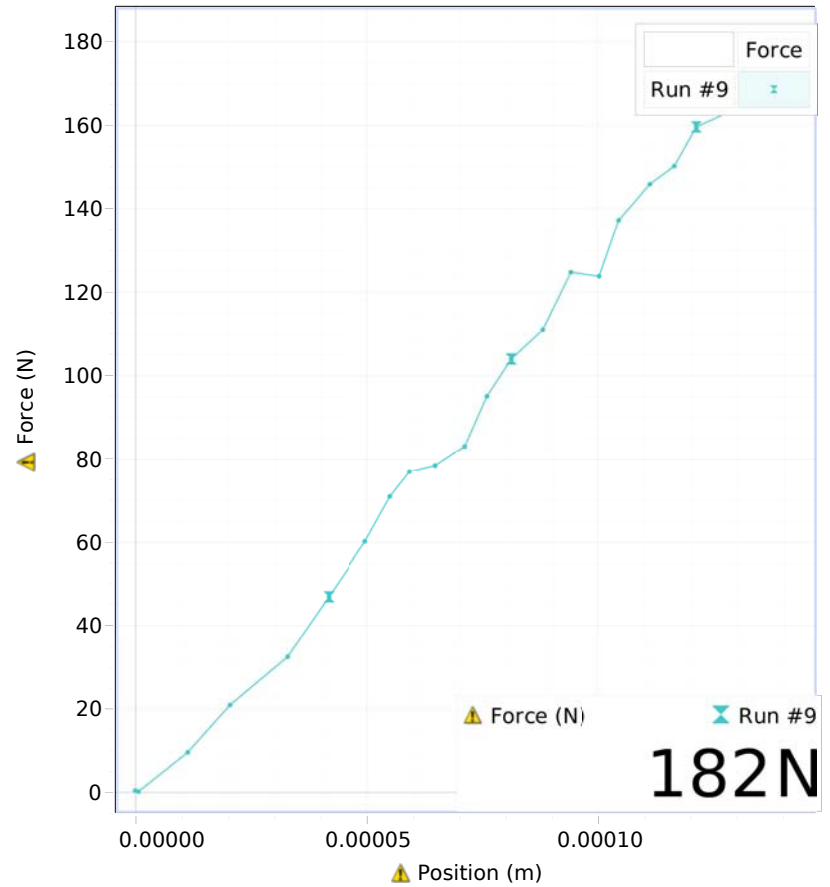
Figure 2. Changing Anvil Spacing

### "Seating" the Test Sample and Setting Pre-Load

1. Place the steel test sample across the support anvils. Turn the crank counter-clockwise until the load anvil is almost touching the sample.
2. Click on Record. Continue cranking counter-clockwise, watching the graph and digits display. Run the applied force up to about 50 N, and then decrease it back to about 10 to 20 N. Try not to let the force go completely to zero.
3. While still recording data, run the force up and down again. It is necessary to load and unload the system several times to remove all the slack and properly "seat" the test sample. When two subsequent curves track on top of each other, you are ready to proceed.
4. With data still being recorded, reduce the force back down to about 20 N, and then Click on Stop. Do NOT change the crank position. Since the sensor will auto-zero next time you record data, this puts a 20 N pre-load on the sample which results in better data. You should use this same method when performing any compliance calibration of the Materials Tester.

Note: Do not exceed the Yield Strength of the material! If the sample is permanently bent, you went too far. For these samples, you only need to apply a max force of 100 to 200 N .

initial L = ? = 8.89 cm  
compliance cal of 500 N cal (more than was needed) with 20 N pre-load



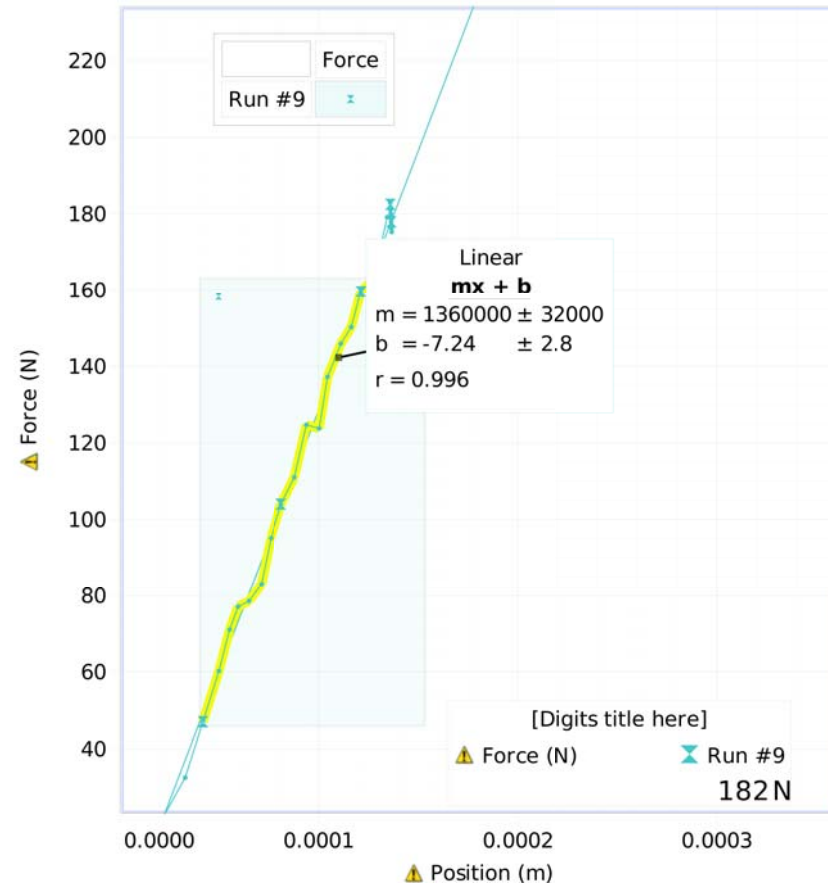
Seating Sample

## Taking Data

1. Click on Record. Turn the crank counter-clockwise, bending the sample. Increase the force to about 100 N.
2. Click on Stop. The data should be fairly linear. It is ok if there is a slight curvature at the beginning or end, but if there is not a straight section in the middle, you probably have something wrong.
3. To take another run of data, turn the crank clockwise to completely remove the tension. Repeat **all** the steps from the previous page to properly re-seat the sample, then take another run of data.
4. Use a linear curve fit to find the slope. This is the stiffness ( $F/\Delta x$ ) of the length of beam you are testing. Record these values in the table below.
5. Decrease the anvil spacing,  $L$ , by about 1 cm and repeat.
6. Measure the stiffness for lengths down to about 3 cm.

Three-Point Bending

	L (m)	F/ $\Delta x$ (N/m)
1	0.0898	63400
2	0.0792	89900
3	0.0715	125000
4	0.0616	188000
5	0.0505	337000
6	0.0418	617000
7	0.0321	1380000
8		

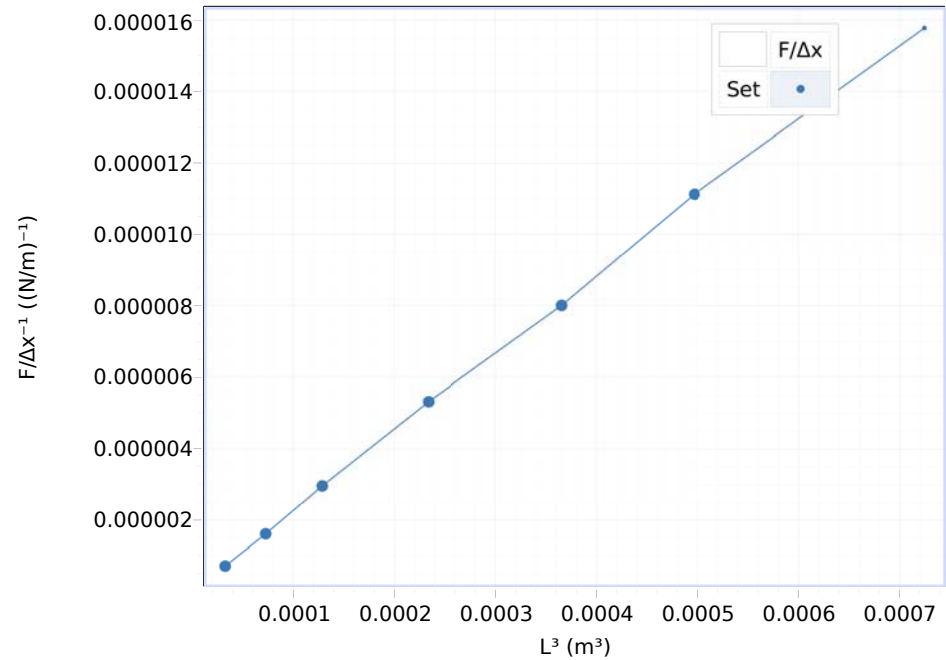


3 Point Bending

### Beam Stiffness and Anvil Separation.

1. How does the stiffness of the beam depend on the length being tested? Does it increase or decrease with length?
2. Try a QuikCalc on the vertical axis to graph  $(F/\Delta x)^{-1}$ .
3. Is this linear? Try a QuikCalc on the horizontal axis to graph the square root of length ( $L^{1/2}$ ).
4. Is this linear? Did it make it more or less linear? Try a QuikCalc on horizontal axis to graph the length squared ( $L^2$ ).
5. Is this linear? Try a QuikCalc on horizontal axis to graph the length cubed ( $L^3$ ).
6. What do you conclude?

	L (m)	F/ $\Delta x$ (N/m)
1	0.0898	63400
2	0.0792	89900
3	0.0715	125000
4	0.0616	188000
5	0.0505	337000
6	0.0418	617000
7	0.0321	1380000
8		
9		



L dependence

#### Answers

Inverse relationship: Increasing L decreases stiffness. But graph not linear  
 $L^{1/2}$ : worse, wrong correction  
 $L^2$ : better, but still not linear  
 $L^3$ : linear. Shows that  $F/\Delta x \propto 1/L^3$

### Three-Point Bending Test

A test sample is supported by two anvils separated by a length,  $L$ , as shown in Figure 3. A load,  $F$ , is applied in the middle, an equal distance from each anvil, and the resulting flexure ( $\Delta x$ ) is measured.

The ratio of  $F/\Delta x$  is the stiffness of the sample, and depends on the length,  $L$ . It also depends on the cross section shape and area of the sample, as well as the material.

If "E" is the Flexural Elastic Modulus for the material, and "I" is the Area Moment of Inertia for the sample, then

$$F/\Delta x = 48IE/L^3 \quad \text{Eqn. (1)}$$

The Area Moment of Inertia depends on the shape of the cross section of the sample. For a round rod of radius,  $r$ ,

$$I_{\text{rod}} = 1/4 \pi r^4 \quad \text{Eqn. (2)}$$

Thus we see from Eqn. (1) that the stiffness ( $F/\Delta x$ ) is inversely proportional to the cube of the anvil separation,  $L$ , and a graph  $F/\Delta x$  vs.  $1/L^3$ , yields a straight line with a slope =  $48IE$ . Finally, solving for E yields

$$E = (\text{slope})/48I \quad \text{Eqn. (3)}$$

Note. The Flexural Modulus is technically not the same as Young's Modulus. Bend testing involves both tensile and compressive stresses, and for some materials these moduli are different.

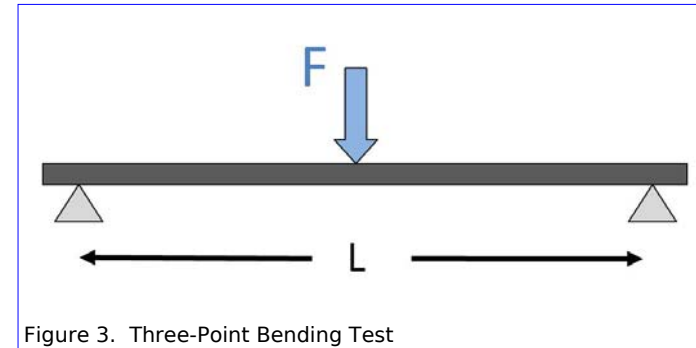


Figure 3. Three-Point Bending Test

### Calculating Moment of Inertia

1. Measure the radius of the test sample.
2. Use Eqn. (2) to calculate the Area Moment of Inertia.

Inertia:

diameter = 3.15 mm

$r = 1.575$  mm

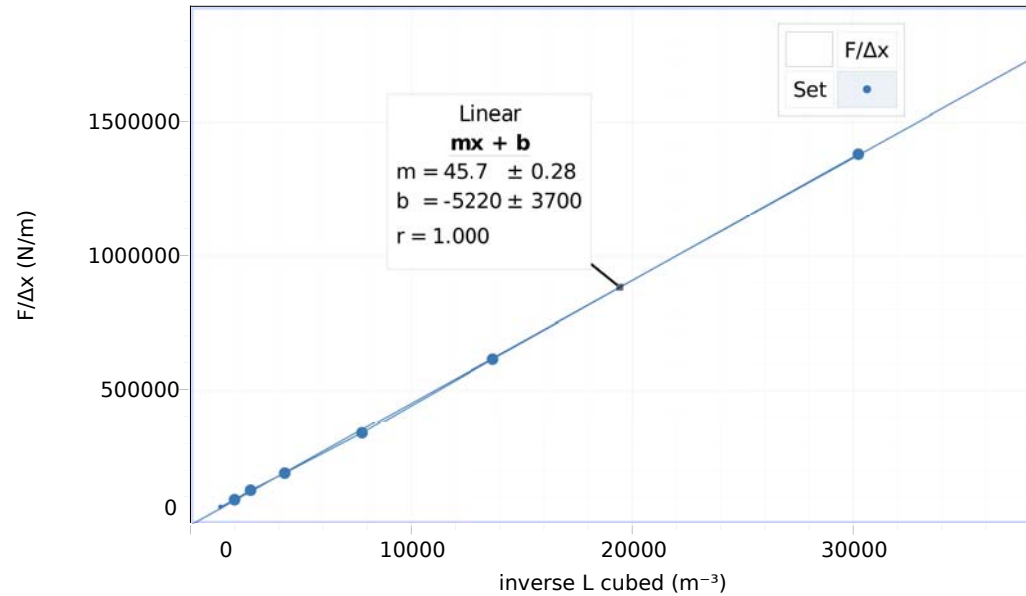
$I = 1/4 \pi r^4 = 1/4 \pi (1.575 \times 10^{-3} \text{m})^4 = 4.833 \times 10^{-12} \text{m}^4$

## Flexural Modulus

1. Note that a calculation of  $1/L^3$  has been added to the table. Confirm that the calculation is correct.
2. Use a linear curve fit to find slope of the graph.
3. Use Eqn. (3) to calculate the Flexural Modulus for your steel sample.
4. For your sample, how does the Flexural Modulus compare to Young's Modulus?

### Three-Point Bending

inverse L cubed = $1/[L(m)]^3$			
	F/ $\Delta x$ (N/m)	L (m)	inverse L cubed ( $m^{-3}$ )
1	63400	0.0898	1380.93
2	89900	0.0792	2012.91
3	125000	0.0715	2735.78
4	188000	0.0616	4278.17
5	337000	0.0505	7764.72
6	617000	0.0418	13692.13
7	1380000	0.0321	30233.25
8			
9			
10			



### Finding the Modulus

Answers:

Slope =  $45.7 \text{ Nm}^2$   
 $E = 197 \text{ GPa}$

Both moduli seem to be the same, around 200 GPa