3. FLUID DYNAMICS

STRUCTURED

Driving Question | Objective

How is the height of a fluid column related to the exit velocity of the fluid at the bottom of the column? Experimentally determine the mathematical relationship between the height of a fluid column and the exit velocity of that fluid.

Materials and Equipment

- Support stand, 10 cm high
- Meter stick
- Water reservoir with a nozzle or hole at the bottom
- Water catch basin
- Pen, felt marker
- Distilled water to fill the water reservoir

Background

As an object falls, it will continue to gain speed (as long as we neglect air resistance). The mathematical relationship between height and speed can be easily determined by using the concepts of gravitational potential energy and kinetic energy. According to the theory of conservation of energy, the total energy of the object will remain the same as the object falls since gravitational potential energy transfers to kinetic energy. We know that this theory works well for moving objects, but what about flowing fluids?

In this activity, you will use a tall reservoir, filled with water, with a nozzle at the bottom. As water exits the nozzle horizontally, the height of the water column decreases. Your goal is to determine the mathematical relationship between the velocity of the water flowing out of the nozzle and the height of the water column.

The height of the water column can be easily measured using a meter stick. Since the velocity of the water exiting the nozzle cannot be measured directly, the velocity will be determined by measuring the range of the projectile water and using kinematic equations.

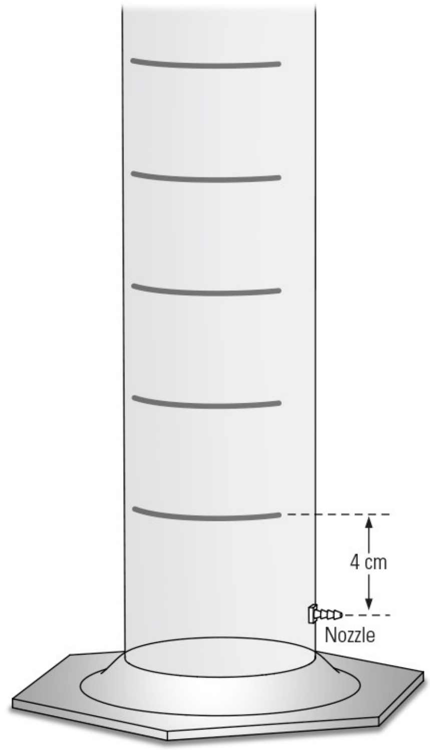
Safety

Follow these important safety precautions in addition to your regular classroom procedures:

- Make necessary arrangements to your workstation to avoid getting water on any electronic equipment.

Procedure

SET UP



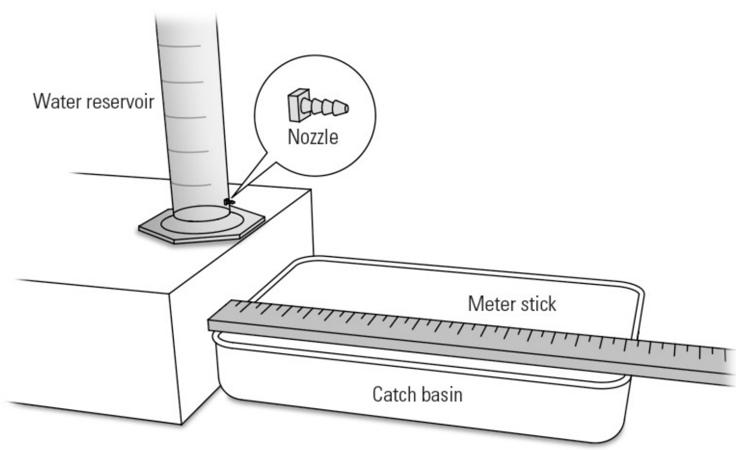
1. Using your meter stick and a felt-tipped pen, measure and mark five 4-cm graduations on the side of the water reservoir starting from zero at the nozzle or hole at the bottom of the reservoir.

2. Lay the catch basin flat on your lab table and set the water reservoir on a stand in front of and approximately 10 cm above the catch basin. Point the nozzle or hole towards the catch basin.

3. Use the meter stick to record the height Δy of the nozzle above the top of the catch basin. Record this value, "Nozzle height," above Table 1 in the Data Analysis section below.

4. Set your meter stick flat on or across the top of the water catch basin so that the length of the meter stick is aligned with the expected path of the stream and next to (but not under) the point where the water enters the catch basin. Align the zero on the meter stick with the end of the nozzle or hole.

5. Make certain the nozzle or hole on the bottom of the water reservoir is plugged or otherwise securely closed. Fill the water reservoir with distilled water 2 cm to 3 cm above the highest graduation mark.



NOTE: Unless you are absolutely certain about the path of the water stream, you may want to open the nozzle or hole and let a small amount of water out to properly align the meter stick with the stream; however, if you do so and the water level in the reservoir falls close to or below the highest graduation mark, add more distilled water to bring the water level 2 cm to 3 cm above the graduation.

COLLECT DATA

6. When the nozzle or hole is opened and water begins to stream out (DO NOT RELEASE THE WATER STREAM YET), perform the following steps to measure the range of the water flow:

   a. One student (student A) will watch the water level in the reservoir as it decreases while another student (student B) records the range of the water stream.

   b. When the water level reaches the first graduation on the side of the container, student A will tell student B to mark or record the range read from the meter stick.

   c. As the water level in the reservoir decreases, repeat the previous step for each graduation on the side of the reservoir.

7. Now open the nozzle or hole and allow the water to pour freely into the catch basin. Record data as described in the previous steps.

8. Once the water level has passed the final graduation mark, close the nozzle or hole and record your range measurements for each height in Table 1.

   NOTE: The highest graduation represents a height of 20 cm while the lowest represents a height of 4 cm.

Data Analysis

Nozzle height (m): ________________________________

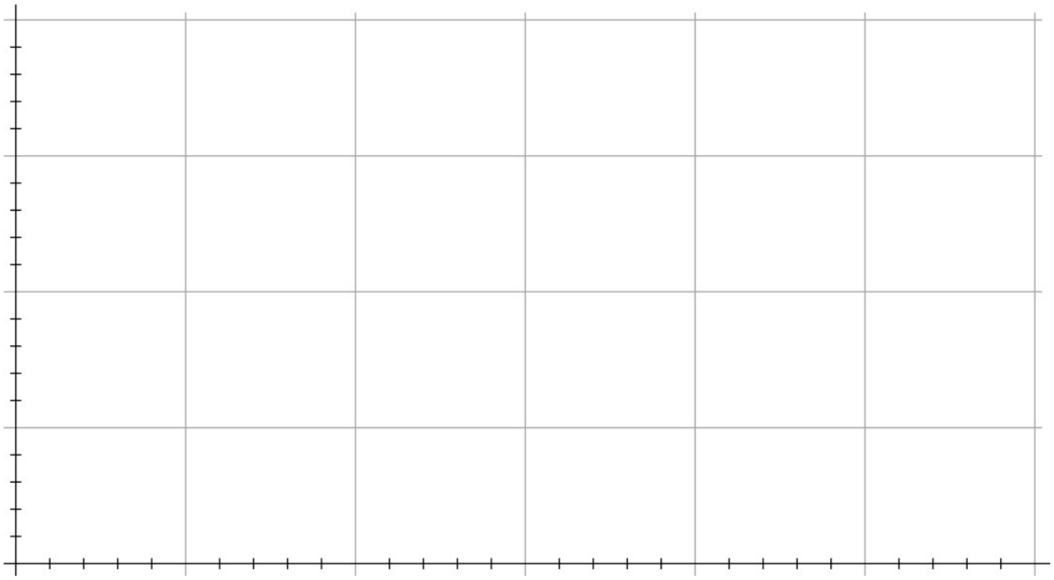
Table 1: Determining the relationship between the height of a column of water and the exit velocity of the water

Water Height (m)	Water Exit Range (m)	Velocity (m/s)	Velocity^2 (m^2/s^2)
0.200			
0.160			
0.120			
0.080			
0.040			

❓ 1. Using the water exit range measurements from Table 1 and the nozzle height, calculate the exit velocity for each range measurement using kinematic equations for projectile motion. Record the results into Table 1. Show your work for one of the calculations.

2. Plot a graph of *velocity* versus *water height* in the blank Graph 1 axes. Be sure to label both axes with the correct scale and units.

Graph 1: Exit velocity versus height of a water column

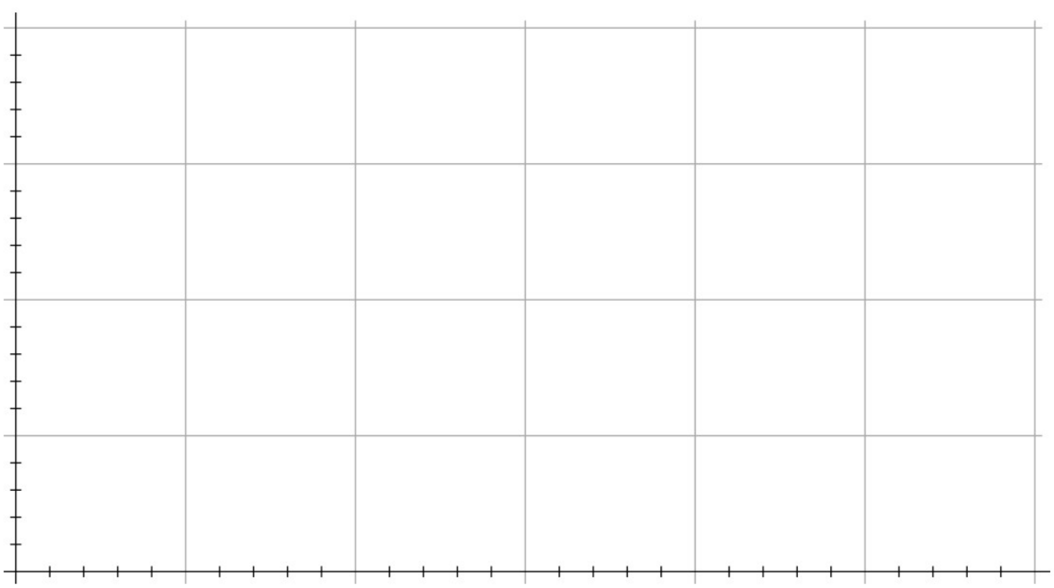


3. How does the exit velocity change as the height of the water column decreases?

4. To produce a linear graph relating the height of the water column and the exit velocity, calculate velocity squared for all of your velocity values and record the results in Table 1.

5. Plot a graph of *velocity squared* versus *water height* in the blank Graph 2 axes. Be sure to label both axes with the correct scale and units.

Graph 2: Exit velocity squared versus water height of a water column



Analysis Questions

1. How is the exit velocity mathematically related to the height of the water column? Use terms such as proportional, inversely proportional, linear, or quadratic in your response.

2. The expression relating potential and kinetic energy in fluids is known as Bernoulli's equation

$$P_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2$$

where P is the pressure in a fluid at height y above the bottom of the fluid container, ρ is the density of the fluid, and v is the velocity of the fluid. The term that includes the height is the potential energy per volume and the term that includes the velocity is the kinetic energy per unit volume. For this experiment, point 1 is at the top of the water column and point 2 is at the end of the nozzle.

a. Assuming that points 1 and 2 are at the same pressure (atmospheric pressure) and approximating the speed of the water column to be 0 m/s, simplify Bernoulli's equation.

b. How does your data support the simplified equation?

c. Using the assumptions mentioned above, calculate the potential energy per volume and the kinetic energy per volume for each data point. Also assume that the density of the water is 1000 kg/m^3. Enter your results in Table 2.

Table 2: Determining the potential energy and kinetic energy of water exiting a nozzle

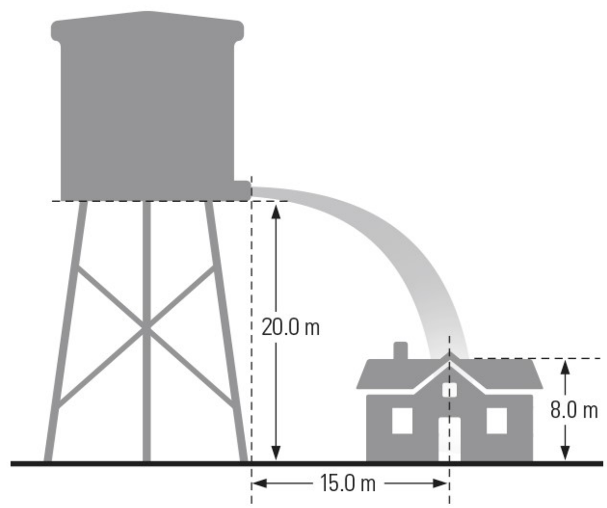
Water Height (m)	Velocity (m/s)	Potential Energy per Volume (J/m^3)	Kinetic Energy per Volume (J/m^3)
0.200			
0.160			
0.120			
0.080			
0.040			

d. According to the simplified Bernoulli equation, the potential energy per volume at point 1 should be equivalent to the kinetic energy per volume at point 2. Do you find that to be the case? If not, what do you account for the discrepancy in your data?

3. What variable in your experiment could you change in order to obtain different velocity^2 versus height graph? Describe a possible way to change this variable (even if you do not have the resources to change this variable in your lab).

Synthesis Questions

1. A city holding tank for water sits 20.0 m above the city. If a house near the holding tank was on fire, to what height would firefighters have to drain the tank so water sprayed from the bottom of the tank hits the top of the house at its center, 8.0 m above the ground and 15 m away? Show your work.



2. Assume the tank from the previous question is sealed airtight with an air gap between the water's surface and the top of the tank. The air pressure in the air gap is 30.4 kPa, while atmospheric pressure outside the tank is 101.4 kPa. Assuming the velocity of the water level in the tank is effectively zero, to what height will firefighters now need to drain the tank so water hits the burning house?