20. Projectile Motion

Driving Questions

What is the relationship between the net force applied to an object and its motion?

How does and object moving in two dimensions differ from and object traveling in only one dimension?

Background

In everyday life, we observe many examples of objects moving in two dimensions: a football punt, a tennis volley, a long fly ball in baseball, and even water projected out of a drinking fountain. An object, without the capacity to glide in unpowered flight, is in projectile motion. Previous study on linear motion investigated constant speed and velocity as well as acceleration. To analyze projectile motion requires us to consider how the projectile moves initially, both in the horizontal and vertical directions. When we perform vector addition on the initial horizontal and vertical velocity components, it gives us a representation for the initial velocity of the projectile. Conversely, if we know the initial velocity of the projectile, we can use trigonometric relationships to get the initial velocity in both the vertical and horizontal directions:

$$v_x = v_0 \cos(\theta) \quad \textbf{Eq.1}$$

$$v_y = v_0 \sin(\theta) \quad \textbf{Eq.2}$$

You will launch the projectile at various angles to determine the relationship between initial velocities and the motion of the projectile.

Materials and Equipment

For each student or group:

- Data collection system
- Digital adapter
- Time of flight pad
- Projectile launcher
- Projectile
- Photogate (2)
- Photogate mounting bracket
- Short rod
- Pencil or pen
- Ram rod
- Plumb bob
- Large table clamp
- Sheet of white paper (10)
- Tape Measure
- Tape, 1 roll
- Digital extension cable (optional)
- Carbon Paper (optional)

Safety

Add these important safety precautions to your normal laboratory procedures:

♦ When using the projectile launcher, do not stand directly in front of the launcher at anytime.

♦ Always use eye protection when using launchers.

Sequencing Challenge

The steps below are part of the Procedure for this lab activity. They are not in the right order. Determine the proper order and write numbers in the circles that put the steps in the correct sequence.

○	○	○	○	○
Collect an initial velocity value for each angle.	Compare your graph of Launch Angle versus Velocity and your graph of Launch Angle versus Distance.	Securely mount the launcher to the table with the large table clamp.	Display Launch Angle versus Velocity in a graph display.	Use the ram rod to load a projectile into the launcher at the maximum setting.

Procedure

After you complete a step (or answer a question), place a check mark in the box (☐) next to that step.

Note: When you see the symbol "�" with a superscripted number following a step, refer to the numbered Tech Tips listed in the Tech Tips appendix that corresponds to your PASCO data collection system. There you will find detailed technical instructions for performing that step. Your teacher will provide you with a copy of the instructions for these operations.

Part 1 - Initial Velocity and Distance

Set Up

1. ☐ Start a new experiment on the data collection system. �(1.2)

2. ☐ Connect the digital adapter to the data collection system. �(2.1)

3. ☐ Connect the photogates to your digital adapter.

4. ☐ Select “velocity between gates” when prompted by your data collection system.

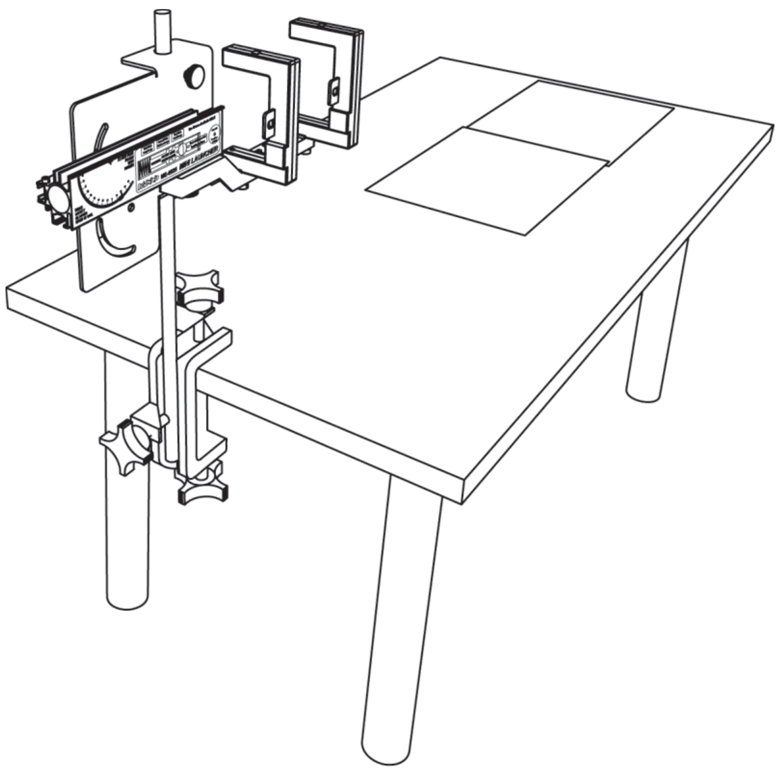
Note: Ensure the space between gates parameter on your data collection system is set to 0.10 m, or the space between your photogates.

5. ☐ Put your data collection system into manual sampling mode with manually entered data. Name the manually entered numerical data Angle measured in degrees. �(5.2.1)

6. ☐ Add a column to the table to display distance in meters. �(7.2.2)

7. ☐ Attach the projectile launcher to a table using the large table clamp and short rod so that the projectiles travel across the longest part of the table.

Note: Make sure that the launcher can rotate through 80° with the photogate bracket mounted.



8. ☐ Place sheets of paper end-to-end in a line across the length of the table in front of the projectile launcher, and secure them in place with tape.

9. ☐ Measure the height from where the point the ball is released to the table top, and record this value in the Data Analysis section.

10. ☐ Use the photogate mounting bracket to mount the photogates to the launcher. Be sure to mount the photogate connected to port 1 of the digital adapter, closest to the launcher.

11. ☐ Set the launcher in the horizontal position with a launch angle of 0°.

12. ☐ Use the ram rod to load a projectile into the launcher, and ensure that the launcher is set to its maximum compression.

Note: If you do not have enough space, use a lower setting on the launcher, but be sure to use the same setting each time.

13. ☐ Launch the projectile, and note the point of impact on the paper.

14. ☐ Lay a sheet of carbon paper on top of the white paper over the point of impact, carbon side down, so that when a ball lands on it there will be a mark on the paper. Place a sheet of paper over the carbon paper to prevent damage to the carbon paper by the projectile.

Note: If carbon paper is not available, look for a small indentation on the paper where the ball hits.

15. ☐ Use the ram rod to load a projectile into the launcher, and ensure that the launcher is set to its maximum compression.

16. ☐ What launch angle do you predict will yield the greatest horizontal range (distance)?

___

___

___

Collect Data

17. ☐ Start a new manually sampled data set. �(6.3.1)

18. ☐ Launch the projectile.

19. ☐ Record a manually sampled Velocity Between Gates data point, and enter the corresponding Angle value. �(6.3.2)

20. ☐ Move the carbon paper, and measure the distance to the mark. Write the Angle next to the mark on the paper.

21. ☐ Loosen the photogate bracket and move the photogates to one side.

Note: When removing and replacing the photogate bracket, be very careful not to block the photogates and to replace the bracket to same position each time.

22. ☐ Use the ram rod to load a projectile into the launcher, and ensure that the launcher is set to its maximum compression.

23. ☐ Put the photogates back in position, and use the angle indicator on the launcher to position the launcher at the next angle, 10°.

24. ☐ Repeat the data collection steps, increasing the angle of inclination by 10° each time until you have recorded a data point every 10° from 0° to 80°.

25. ☐ Stop the manually sampled data set. �(6.3.3)

26. ☐ Measure and enter the horizontal distance for each Angle value into the table on your data collection system, and then copy all the values to Table 1 in the Data Analysis section.

Analyze Data

27. ☐ Display Distance on the *y*-axis of a graph with Angle on the *x*-axis. �(7.1.1)

28. ☐ Sketch the graph of Distance versus Angle in the Data Analysis section.

29. ☐ Display Velocity on the y-axis of a graph with Angle on the x-axis. �(7.1.1)

30. ☐ Sketch the graph of Distance versus Angle in the Data Analysis section.

31. ☐ Save your experiment as instructed by your teacher. �(11.1)

Part 2 - Time of Flight

Set Up

32. ☐ Start a new experiment on your data collection system. �(1.2)

33. ☐ Remove the photogate furthest from the launcher from the photogate mounting bracket.

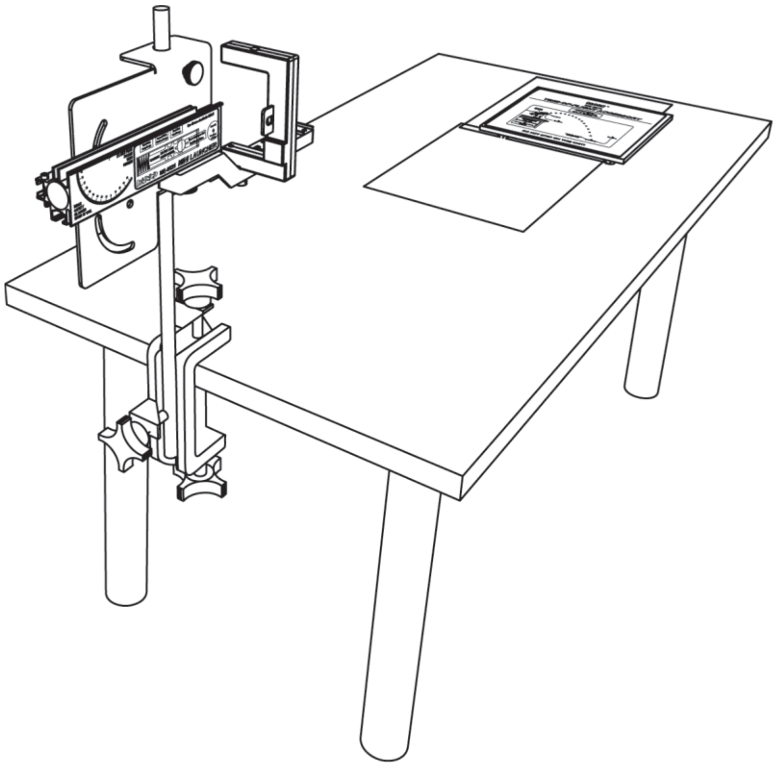
34. ☐ Disconnect this photogate from port 2 of your digital adapter.

35. ☐ Connect the Time of Flight pad to port 2 of your digital adapter.

36. ☐ Select Time of flight when prompted by your data collection system.

37. ☐ Put your data collection system into manually sampling mode with manually entered data. Name the manually entered numerical data "Angle" measured in degrees. �(5.2.1)

38. ☐ Position the time of flight pad over the 0° impact point.



39. ☐ Set the launcher in the horizontal position with a launch angle of 0°.

40. ☐ Use the ram rod to load a projectile into the launcher, and ensure that the launcher is set to its maximum compression.

41. ☐ Which angle do you think will yield the greatest time of flight value?

____

____

____

Collect Data

42. ☐ Start a new manually sampled data set �(6.3.1)

43. ☐ Launch the projectile.

44. ☐ Record a manually sampled data point, and enter the Angle value. �(6.3.2)

45. ☐ Move the time of flight pad to the next mark in the series.

46. ☐ Loosen the photogate bracket, and move the photogate to one side.

47. ☐ Use the ram rod to load a projectile into the launcher, and ensure that the launcher is set to its maximum compression.

48. ☐ Put the photogates back into position, and use the angle indicator on the launcher to position the launcher at the next angle, and repeat the data collection steps until you have recorded a data point every 10° from 0° to 80°.

49. ☐ Stop the manually sampled data set. �(6.3.3)

50. ☐ Copy all the values in your table to Table 1 in the Data Analysis section.

Analyze Data

51. ☐ Display Time of Flight on the *y*-axis of a graph Angle on the *x*-axis. �(7.1.1)

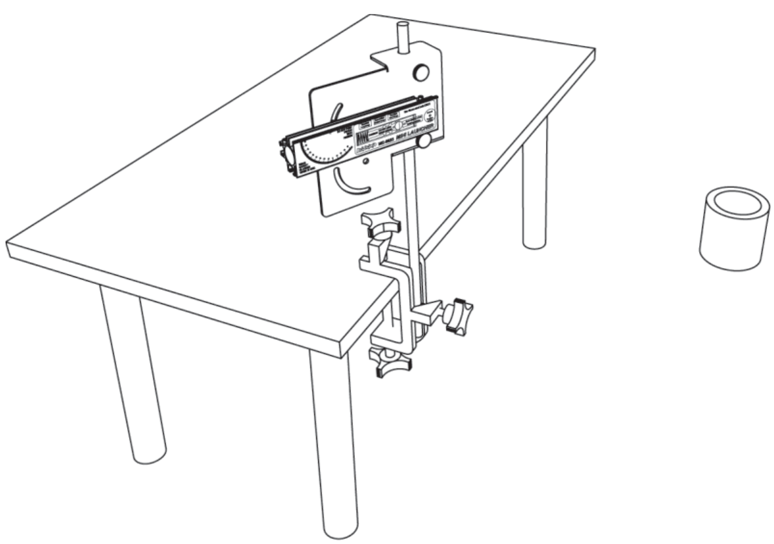
52. ☐ Sketch the graph of Time of Flight versus Angle in the Data Analysis section.

53. ☐ Save your experiment as instructed by your teacher. �(11.1)

Part 3 - The Challenge

54. ☐ Remove the photogate bracket from the launcher.

55. ☐ Move the launcher to the front of your table and into a position so that it will launch to the floor.



56. ☐ Ask your teacher to place a target on the floor in front of the launcher.

57. ☐ Using a plumb bob and measuring tape, measure the vertical and horizontal distance to the target.

58. ☐ Calculate the angle to set your launcher at in order to hit the target, and record this value below.

Note: You may want to find the average of your initial velocities. There is also a trigonometric identity that might help: $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$.

Angle of launcher:_____

59. ☐ When you are ready, aim your launcher, and inform your teacher.

60. ☐ Launch the projectile.

61. ☐ How close were you to the target?

____

____

Data Analysis

Height of the launcher:_____

Table 1: Projectile data

Angle (°)	Velocity (m/s)	Distance (m)	Time of Flight (s)

1. ☐ Choose one of your angles other than 0° , and draw to scale a vector diagram for your projectile at the launch position showing both the horizontal and vertical component velocities. Show how you determine the average initial velocity, and draw the net vector.

2. ☐ Calculate the horizontal and vertical components of velocity for each angle, and fill in the columns in Table 2.

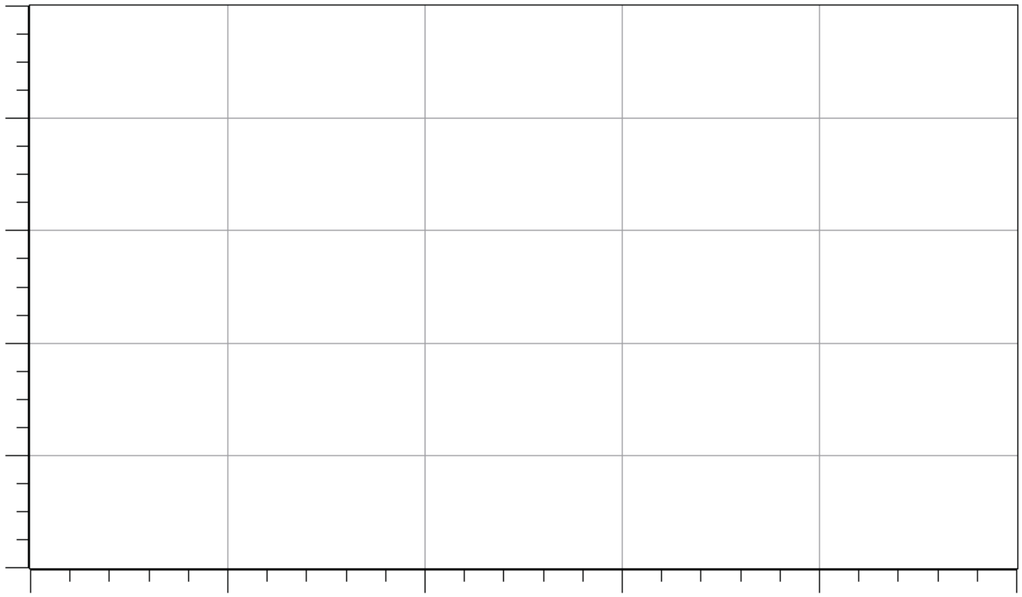
3. ☐ Use the horizontal distance and time of flight to calculate the average horizontal velocity for each angle, and then fill in the corresponding column in Table 2.

4. ☐ Use the vertical velocity and the height of the launcher for each launch angle to calculate the theoretical time of flight of an object shot straight up. Record the result in Table 2.

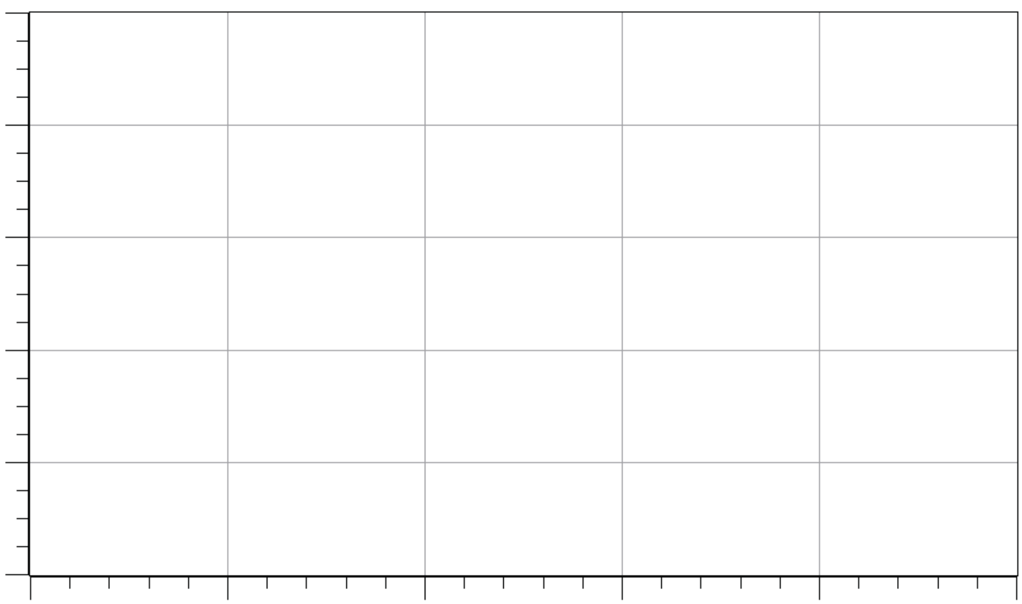
Table 2: Projectile data calculations

Angle (°)	Horizontal Velocity (m/s)	Vertical Velocity (m/s)	Average Horizontal Velocity (m/s)	Theoretical Time of Flight (s)

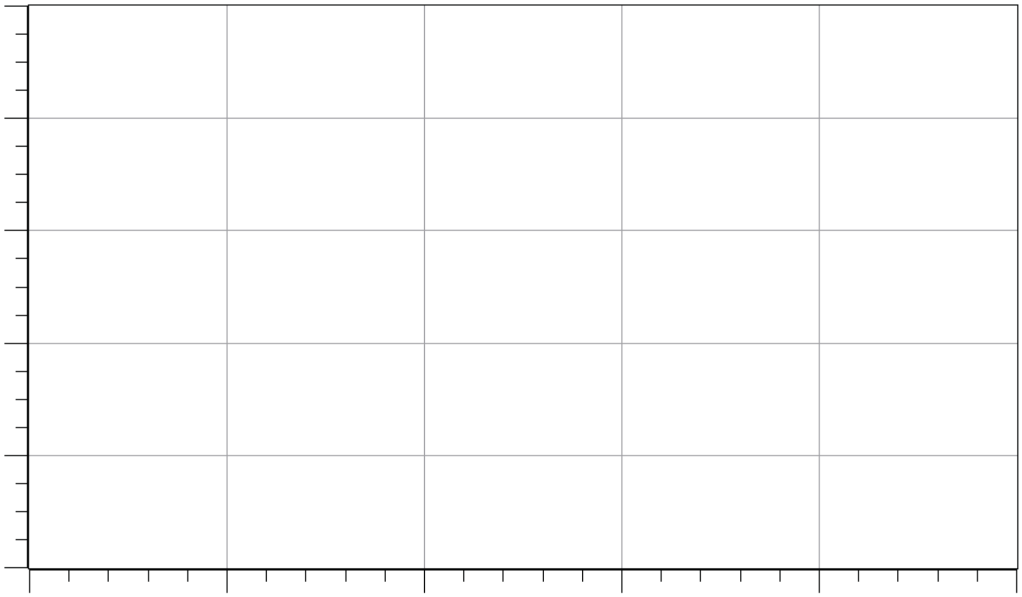
Distance versus Angle



Initial Velocity versus Angle



Time of Flight versus Angle



Analysis Questions

1. How did the measured horizontal velocities compare to the average horizontal velocities?

2. For any projectile launched horizontally, what can you state about the horizontal velocity? Is it constant or does it change over time?

3. Which launch angle will yield the maximum horizontal range?

4. Which launch angles yield the same results for the horizontal range?

Synthesis Questions

Use available resources to help you answer the following questions.

1. An astronaut on the moon has a small launcher that lets him pass tools to his partner at the same height on the other side of a crater floor. The initial velocity of the launcher is 15 m/s, and his partner is 30 m away. If the gravity on the moon is 1/6 that of Earth, what angle gives the shortest travel time?

Multiple Choice Questions

Select the best answer or completion to each of the questions or incomplete statements below.

1. What do we call any object fired, launched, or kicked into unpowered flight?

- **A.** Trajectory
- **B.** Projectile
- **C.** Projectory
- **D.** Trajectile

2. The magnitude of the horizontal velocity component of a projectile is based upon which two variables?

- **A.** Distance and time
- **B.** Speed and time
- **C.** Acceleration and time
- **D.** Gravity and time

3. The magnitude of the vertical velocity component of a projectile is based upon which two variables?

A. Distance and time

B. Speed and time

C. Acceleration and time

D. Gravity and time

4. Which of the following pair of angles does not yield the same horizontal range?

A. 22 and 68 degrees

B. 43 and 47 degrees

C. 18 and 72 degrees

D. 33 and 59 degrees

Key Term Challenge

Fill in the blanks from the list of randomly ordered words in the Key Term Challenge Word Bank.

1. When a ball falls vertically to the floor, we say it is in a ____________ state of motion. When we launch a ____________ horizontally, its horizontal velocity is ____________. The shape of the ball's flight is called a trajectory and is parabolic in shape. The maximum ____________ of flight depends on the launch angle and the launch ____________. The force due to ____________ directly relates to the free fall acceleration of the ball.

Key Term Challenge Word Bank

Paragraph 1

Velocity

Acceleration

Range

Free fall

Constant

Trajectory

Gravity

Force

Inertia

Projectile

Newton's second law