NAME PERIOD DATE

12. PERIODIC MOTION: MASS AND SPRING

STRUCTURED

Driving Question | Objective

What variables affect the period of oscillation of a mass and spring system? Experimentally determine the physical properties of a hanging mass and spring system that affect its period of oscillation.

Materials and Equipment

- Data collection system
- Table clamp or large base
- Support rod, 60-cm or taller
- Support rod, 45-cm
- Right angle clamp
- Meter stick
- Hooked mass set
- PASCO Motion Sensor[1]
- Springs of similar size (diameter and length), but varying spring constant (3), 1–15 N/m
- Springs with similar spring constant and diameter, but of varying length (2), 0.1–0.3 m
- Tape

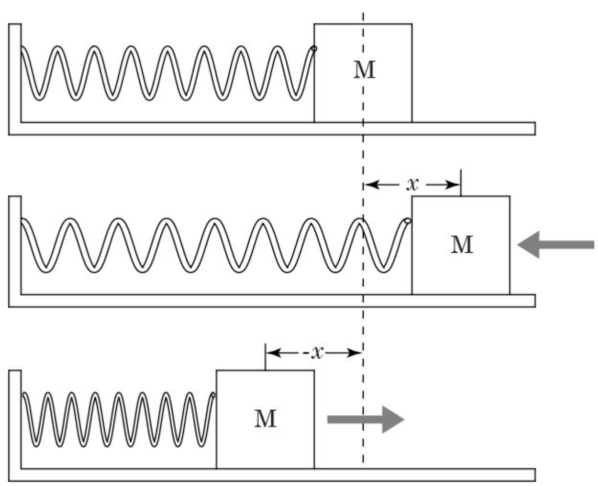
[1]www.pasco.com/ap18

PASCO Motion Sensor

Background

It is easy to think of examples of oscillating objects in our physical world. A bungee jumper, the swinging pendulum of a clock, and the vibrating string of a guitar are all examples of oscillations around us. By definition, oscillating objects move back and forth in some form of repetitive motion, so they are *periodic,* which means that although the oscillations may be small or large, the time for each individual cycle of motion is the same.

As an object oscillates, it must pass through its original (or *undisturbed*) position, called the *equilibrium point*. The time for one complete cycle of oscillation is called the *period* and the maximum distance the oscillating object moves from the equilibrium point is referred to as the *amplitude*.



What makes these objects move in a repeating pattern? Let us look at an oscillating mass attached to a spring, sliding along a surface of negligible friction.

If the spring is not stretched, it does not apply any force to the mass. However, when the spring (with spring constant k) is stretched or compressed, displacing the mass some distance $\vec{x}$, it will apply a force $\vec{F}_s$ proportional to the displacement of the mass but in the opposite direction of that displacement (Hooke's Law):

$$\left|\vec{F}_s\right| = -k\left|\vec{x}\right| \qquad (1)$$

After the mass is displaced, if left to move freely, it will accelerate toward its equilibrium point until it passes through that position, after which it will experience a restoring force applied by the spring, drawing it back toward the equilibrium position once again. This cycle repeats itself and the result is a periodic motion.

In this lab activity, you will explore what variables affect the period of oscillation of a mass and spring system; but unlike the example above, you will use a system oriented vertically to avoid external forces other than gravity acting on the system.

RELEVANT EQUATIONS

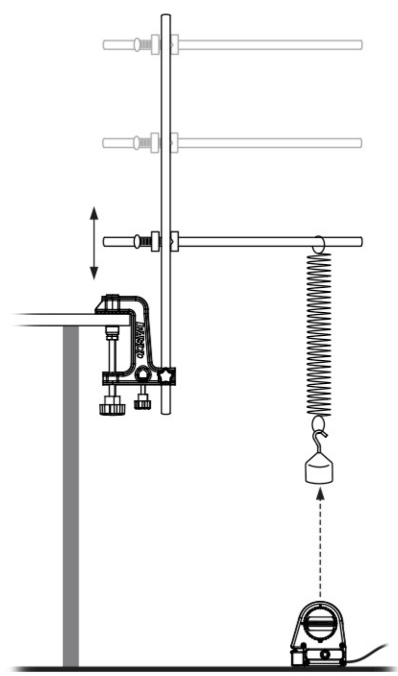
$$\left|\vec{F}_s\right| = -k\left|\vec{x}\right| \tag{1}$$

This equation states that a displaced spring will deliver a restoring spring force $\vec{F}_s$ proportional to the displacement $\vec{x}$ but in the opposite direction. The constant of proportionality k is known as the *spring constant*.

Procedure

Part 1 – Displacement and Period

SET UP



1. Mount the long support rod to the table clamp or large base, and then attach the smaller support rod at a right angle using the right angle clamp. The horizontal support rod should extend past the edge of the lab table.

2. Choose one spring and attach it to the horizontal support rod so it hangs beyond the edge of the lab table. Place a small piece of tape over the spring where it attaches to the rod to prevent it from sliding.

3. Hang 200 g of mass from the spring, and then place the motion sensor on the lab table or floor directly under the hanging mass with the front of the sensor aimed up at the bottom of the mass. Set the switch on the top of the sensor to the cart icon.

4. Adjust the height of the mass and spring so that the mass hangs, motionless, approximately 50 cm above the motion sensor.

5. Connect the motion sensor to the data collection system and then create a graph display of position versus time.

COLLECT DATA

6. Allow the mass to hang motionless from the spring, and then use the meter stick to note the height of the mass; this height is the equilibrium position of the mass and spring system.

7. Raise the mass 4 cm vertically from the equilibrium position and release it to oscillate freely.

8. After the mass has begun oscillating, start recording data.

9. Once the system has finished at least 10 complete oscillations, stop recording data.

10. Use the tools on your data collection system to determine the time needed to complete 10 oscillations. Record this value, as well as the initial vertical displacement, into Table 1 in the Data Analysis section.

11. Repeat the same data collection steps 2 more times, increasing the initial vertical displacement by an additional 4 cm each trial. Record the time for 10 complete cycles and the initial vertical displacement for each trial into Table 1.

Part 2 – Length and Period

SET UP

12. Choose two springs with the same spring constant but different lengths, and attach one to the horizontal support rod (after removing the spring used in Part 1) so it hangs beyond the edge of the lab table. Place a small piece of tape over the spring where it attaches to the rod to prevent it from sliding.

13. Hang 200 g of mass from the spring, and then adjust the position of the motion sensor so it is directly under the hanging mass with the front of the sensor aimed up at the bottom of the mass.

14. Adjust the height of the mass and spring so that the mass hangs, motionless, approximately 50 cm above the motion sensor.

COLLECT DATA

15. Allow the mass to hang motionless from the spring, and then use the meter stick to note the height of the mass; this height is the equilibrium position of the mass and spring system.

16. Raise the mass 10 cm vertically from the equilibrium position and release it to oscillate freely.

17. After the mass has begun oscillating, start recording data, and then stop after you have recorded at least 10 complete oscillation cycles.

18. Use the tools on your data collection system to determine the time needed to complete 10 oscillations. Record this value into Table 2 in the Data Analysis section.

19. Remove the mass from the spring and measure the unstretched length of the spring. Record this value into Table 2.

20. Repeat the same data collection steps with the second spring. Record the time for 10 complete cycles and the unstretched length of the second spring into Table 2.

Part 3 – Spring Constant and Period

SET UP

21. Choose three springs with the same length but different spring constants, and attach one to the horizontal support rod so it hangs beyond the edge of the lab table. Place a small piece of tape over the spring where it attaches to the rod to prevent it from sliding.

22. Hang 300 g of mass from the spring, and then adjust the position of the motion sensor so it is directly under the hanging mass with the front of the sensor aimed up at the bottom of the mass.

23. Adjust the height of the mass and spring so that the mass hangs, motionless, approximately 50 cm above the motion sensor.

COLLECT DATA

24. Allow the mass to hang motionless from the spring, and then use the meter stick to note the height of the mass; this height is the equilibrium position of the mass and spring system.

25. Raise the mass 10 cm vertically from the equilibrium position and release it to oscillate freely.

26. After the mass has begun oscillating, start recording data, and then stop after you have recorded at least 10 complete oscillation cycles.

27. Use the tools on your data collection system to determine the time needed to complete 10 oscillations. Record this value into Table 3 in the Data Analysis section.

28. Record the spring constant for the first spring in Table 3 (spring constant values will be provided by your teacher).

29. Repeat the same data collection steps with the other two springs. Record the time for 10 complete cycles and the spring constant for each spring into Table 3.

Part 4 – Mass and Period

SET UP

30. Choose one spring with a spring constant greater than 6 N/m, but less than 10 N/m, and attach it to the horizontal support rod so it hangs beyond the edge of the lab table. Place a small piece of tape over the spring where it attaches to the rod to prevent it from sliding.

31. Hang 200 g of mass from the spring, and then adjust the position of the motion sensor so it is directly under the hanging mass with the front of the sensor aimed up at the bottom of the mass.

32. Adjust the height of the mass and spring so that the mass hangs, motionless, approximately 50 cm above the motion sensor.

COLLECT DATA

33. Allow the mass hanging from the spring to become motionless, and then use the meter stick to note the height of the mass; this height is the equilibrium position of the mass and spring system.

34. Raise the mass 10 cm vertically from the equilibrium position and release it to oscillate freely.

35. After the mass has begun oscillating, start recording data, and then stop after you have recorded at least 10 complete oscillation cycles.

36. Use the tools on your data collection system to determine the time needed to complete 10 oscillations. Record this value and the amount of mass hanging from the spring in Table 4 in the Data Analysis section.

37. Repeat the same data collection steps 4 more times, increasing the amount of hanging mass by 100 g each trial. Record the time for 10 complete cycles and the hanging mass value for each trial into Table 4.

Data Analysis

Part 1 – Displacement and Period

Table 1: Period and displacement data for a mass and spring system

Trial	Time for 10 Cycles (s)	Initial Vertical Displacement (cm)	Average Period (s)
1			
2			
3			

1. Calculate the average period for each Part 1 trial in Table 1. Record your results for each trial in Table 1.

$$\text{Average period} = \frac{\text{Time for 10 cycles}}{10}$$

2. Did changing the displacement of the mass affect the period of the mass and spring system? Justify your answer.

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Part 2 – Length and Period

Table 2: Period of two springs with the same spring constant but different length

Spring	Time for 10 Cycles (s)	Spring Length (cm)	Average Period (s)
1			
2			

3. Calculate the average period for each spring. Record your results in Table 2.

4. Did changing the length of the spring affect the period of the mass and spring system? Justify your answer.

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Part 3 – Spring Constant and Period

Table 3: Period of three springs with the same length but different spring constant

Spring	Time for 10 Cycles (s)	Spring Constant (N/m)	Average Period (s)	$\frac{1}{\sqrt{\text{Spring Constant}}}$ $[(\text{N/m})^{-1/2}]$
1				
2				
3				

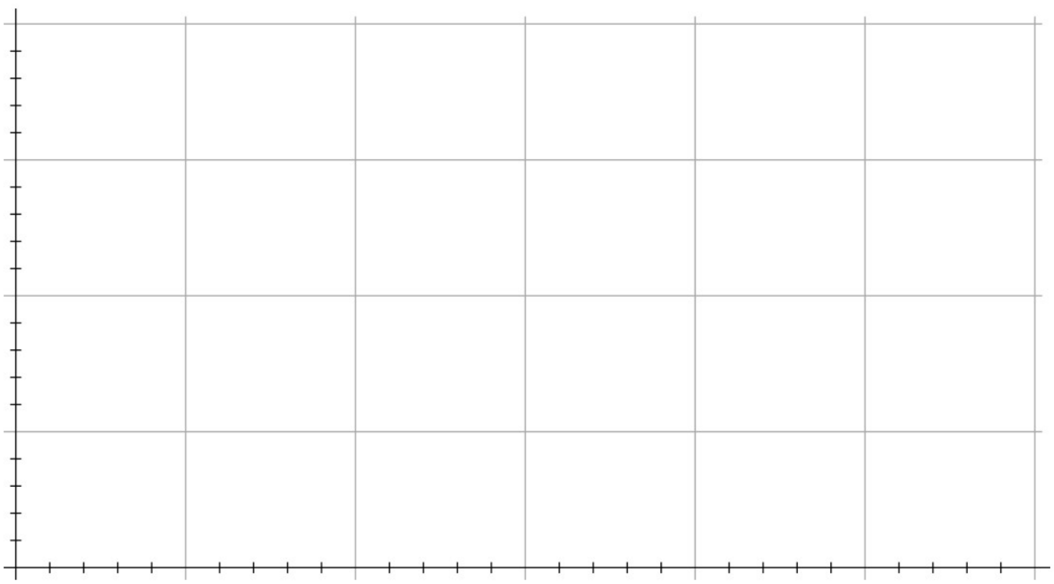
5. Calculate the average period for each spring. Record your results for each trial in Table 3.

6. Did changing the spring constant affect the period of the mass and spring system? Justify your answer.

7. Calculate $1/\sqrt{\text{spring constant}}$ for each spring in Table 3. Record the results in Table 3.

8. Plot a graph of *average period* versus $1/\sqrt{\textit{spring constant}}$ in the blank Graph 1 axes. Be sure to label both axes with the correct scale and units.

Graph 1: Average period versus $1/\sqrt{\text{spring constant}}$ for three mass and spring systems with constant length and mass



9. Based on Graph 1, what is the relationship between period and spring constant for an oscillating mass and spring system (proportional, inverse, squared, et cetera)? Justify your answer.

Part 4 – Mass and Period

Table 4: Period of a mass and spring system with varying mass

Trial	Time for 10 Cycles (s)	Hanging Mass (kg)	Average Period (s)	$\sqrt{\text{Hanging Mass}}$ ($kg^{1/2}$)
1				
2				
3				
4				
5				

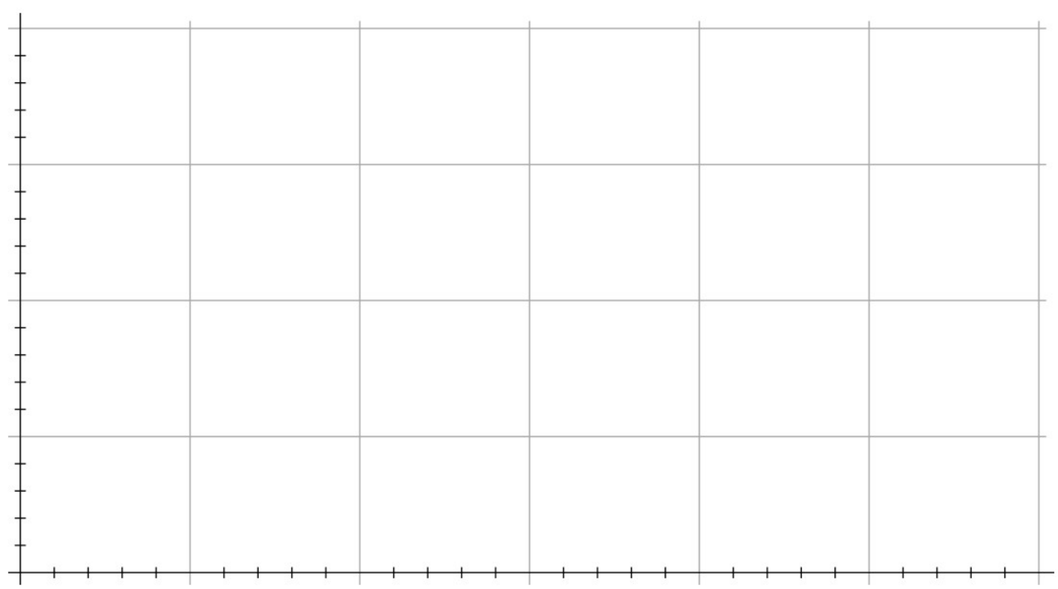
10. Calculate the average period for each spring. Record your results for each trial in Table 4.

11. Did changing the mass affect the period of the mass and spring system? Justify your answer.

12. Calculate $\sqrt{\text{hanging mass}}$ for each trial in Table 4. Record the results in Table 4.

13. Plot a graph of *average period* versus $\sqrt{\textit{hanging mass}}$ in Graph 2. Be sure to label both axes with the correct scale and units.

Graph 2: Average period versus $\sqrt{\text{hanging mass}}$ for a mass and spring system



14. Based on Graph 2, what is the relationship between period and mass for an oscillating mass and spring system (proportional, inverse, squared, et cetera)? Justify your answer.

Analysis Questions

❷ 1. For each part of your experiment, list each variable involved and state whether it was held constant, increased, or decreased.

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❷ 2. In your experiment, what variables (physical properties) affected the period of a mass and spring system and how did they affect the period?

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❷ 3. The mathematical equation describing the period T_s of a mass and spring system is:

$$T_s = 2\pi\sqrt{\frac{m}{k}} \qquad (2)$$

where k is the spring constant of the spring, and m is the amount of hanging mass. Does your data support this mathematical relationship? Justify your answer.

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Synthesis Questions

❷ 1. The motion of oscillating mass and spring systems follow cyclical patterns, so their motion is often described using sinusoidal functions with an angular velocity ω:

$$\omega = \frac{2\pi}{T_s} \qquad (3)$$

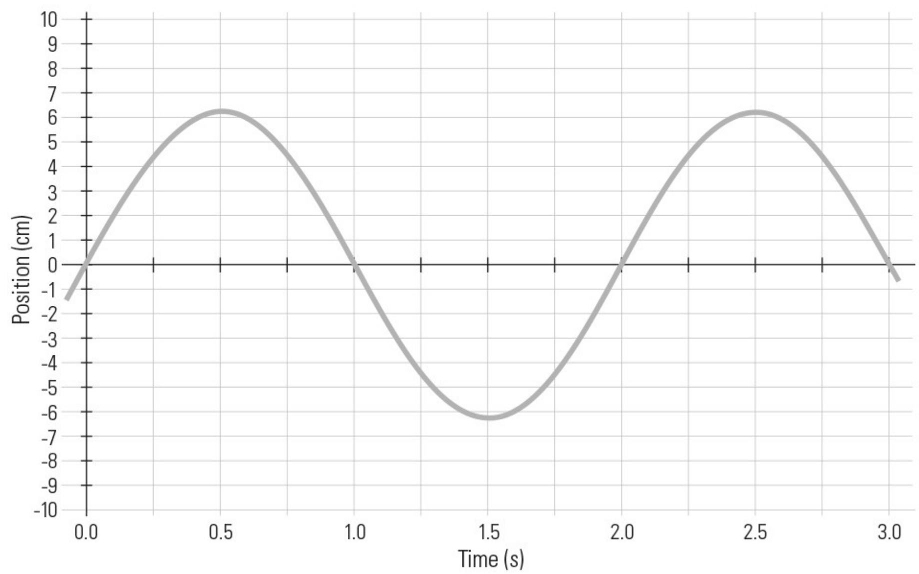
What is the period for a mass and spring system whose angular velocity is 6.28 rad/s? Show calculations and all work.

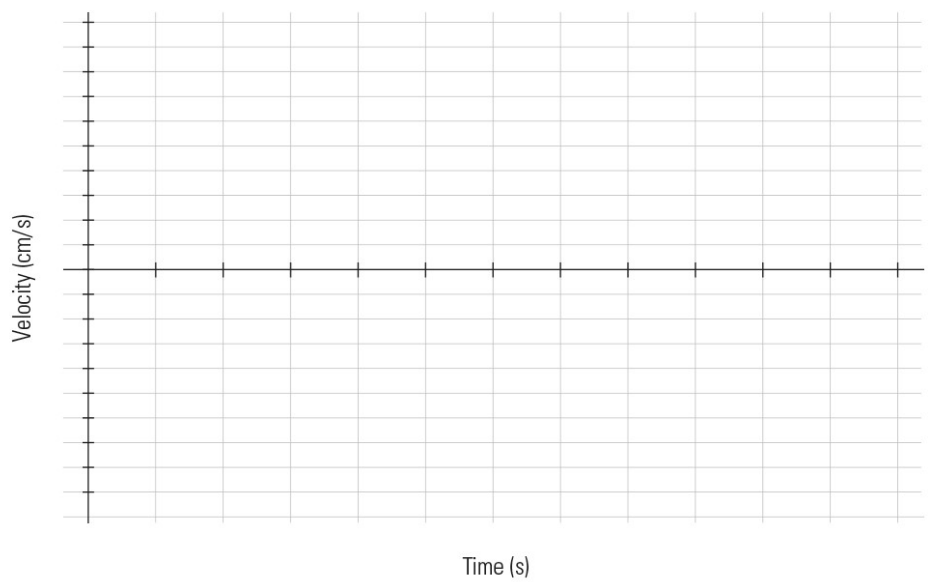
❷ 2. Use Equations 2 and 3 to derive a new expression for ω using just mass m and the spring constant k. Show your work here.

3. The position versus time graph below shows the motion of an oscillating mass and spring system. This graph can be described using the equation:

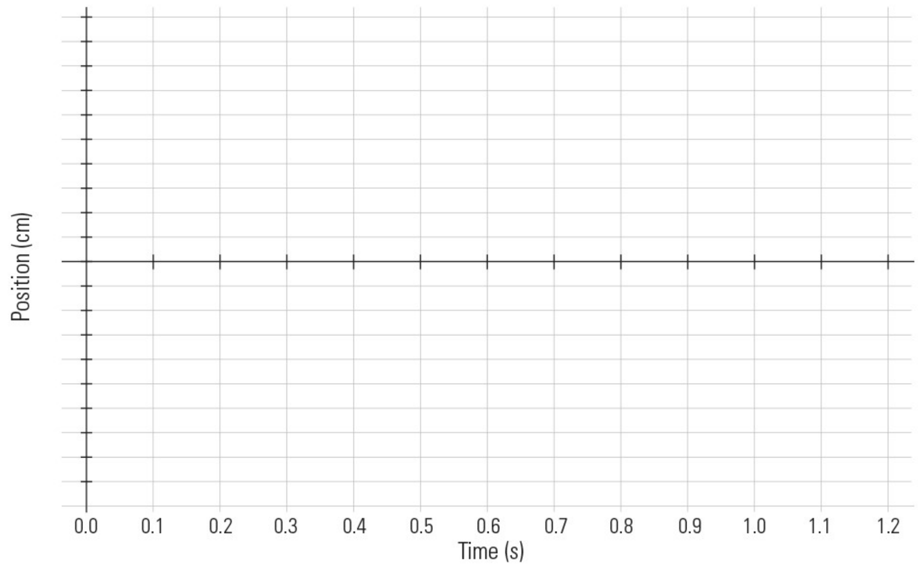
$$x(t) = A\sin\left(\frac{2\pi}{T_s}t\right)$$

where A is the maximum displacement of the mass from equilibrium (both positive and negative). Use your knowledge of the graphical connection between position versus time and velocity versus time graphs to sketch the system's corresponding velocity versus time graph in the blank axes below. Be sure to label both axes with a correct scale.





4. Sketch the position versus time graph for an oscillating mass and spring system whose position at time $t = 0$ is equal to its maximum displacement of 8.0 cm, and takes 0.80 seconds to complete one cycle of motion. Sketch as much of the graph that will fit in the blank axes below, and identify on your sketch the points at which the system has maximum velocity.



5. If the spring in the previous question has a spring constant of 25 N/m, what is the value of the mass? Show calculations and all work.

6. A given spring has a spring constant k and period T_s. If you doubled the mass, what would the new period T_s' be? (Show all work, and put T_s' in terms of T_s).