6. CONSERVATION OF MECHANICAL ENERGY

STRUCTURED

Driving Question | Objective

How do the potential and kinetic energies of an object in a closed system change as its motion changes due to a conservative force? Design an experiment to explore how a cart's kinetic energy, gravitational potential energy, and total mechanical energy change as it rolls down an inclined track.

Materials and Equipment

- Data collection system
- PASCO Smart Gate photogate[1]
- PASCO Photogate Bracket[1]
- PASCO Dynamics Cart Picket Fence[2]
- Table clamp or large base
- Support rod, 45-cm
- Meter stick
- PASCO Dynamics Track[3]
- PASCO Dynamics Track Rod Clamp[4]
- PASCO Dynamics Track End Stop[5]
- PASCO Dynamics Cart[6]
- PASCO Angle Indicator[7]
- Balance, 0.1-g resolution, 2,000-g capacity (1 per class)

[1]www.pasco.com/ap21
PASCO Smart Gate

[2]www.pasco.com/ap16
PASCO Dynamics Cart Picket Fence

[3]www.pasco.com/ap08
PASCO PAStrack

[4]www.pasco.com/ap17
PASCO Dynamics Track Rod Clamp

[5]www.pasco.com/ap11
PASCO Dynamics Track End Stop

[6]www.pasco.com/ap07
PASCO PAScar

[7]www.pasco.com/ap14
PASCO Angle Indicator

Background

Mechanical energy is described as the energy associated with an object's motion and position. A free falling object on earth, isolated from any outside influences, experiences two forms of energy that both contribute to the object's total mechanical energy: gravitational potential energy U_g, and kinetic energy K. The object's total mechanical energy E is equal to the sum of its gravitational potential energy and kinetic energies:

$$E = U_g + K \quad (1)$$

Gravitational potential energy (GPE) is described as the energy stored in an object due to its position in a gravitational field. In the case of a free falling object with mass m, gravitational potential energy is due only to the attraction from earth's gravitational field g, and is based on the height y the object is from earth's surface:

$$U_g = mgy \quad (2)$$

Because the falling object is in the presence of a net conservative force (gravity), it experiences acceleration related to Newton's Second Law as it falls. As the object's speed increases due to this acceleration, its kinetic energy K increases in the form:

$$K = \frac{1}{2}mv^2 \quad (3)$$

Imagine that this object was initially at rest, then lifted to some height y above the ground and released. At the top of the object's fall, the kinetic energy would initially be zero because its speed is zero, but the object would have a non-zero mechanical energy because its potential energy is non-zero. Once the object was released, the kinetic energy would increase as it falls, and keep increasing until it hits the ground, while its gravitational potential energy decreases as it falls, and is equal to zero when it hits the ground.

Relevant Equations

$$U_g = mgy \quad (2)$$

$$K = \frac{1}{2}mv^2 \quad (3)$$

$$E = U_g + K = mgy + \frac{1}{2}mv^2$$

Safety

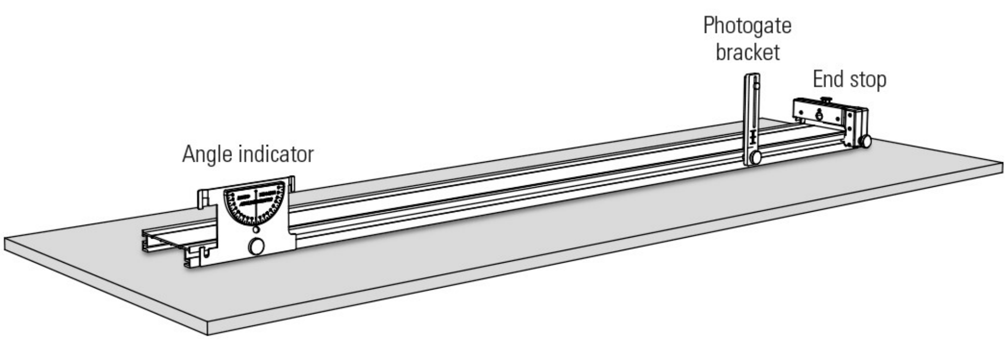
Follow this important safety precaution in addition to your regular classroom procedures:

- The cart can roll off the end of the track and possibly knock objects off the lab bench. Make certain a member of your group catches the cart as it rolls to the end of the track.

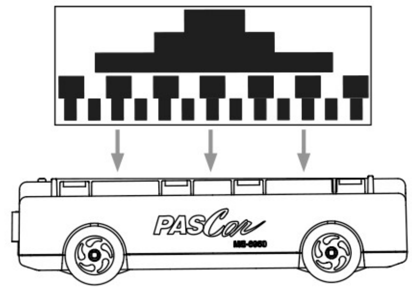
Procedure

Set Up

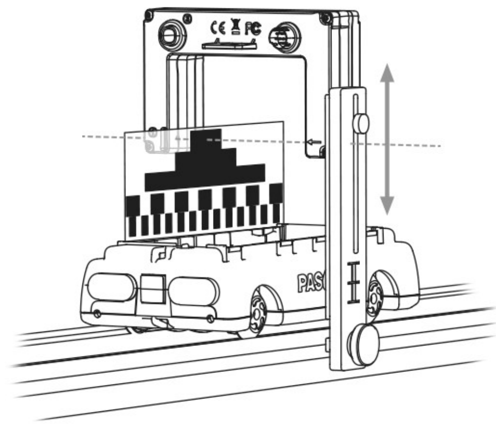
1. Attach the angle indicator, end stop, and photogate bracket to the dynamics track as in the picture below. The end stop should be at the very end of the track and the photogate bracket should be 20 cm from the end stop.



2. Insert the dynamics cart picket fence into the recessed slots on the top of the dynamics cart, oriented with the 2.5-cm solid band as the top-most pattern on the picket fence, and then place the cart on the track.



Insert picket fence into slot in top of dynamics cart

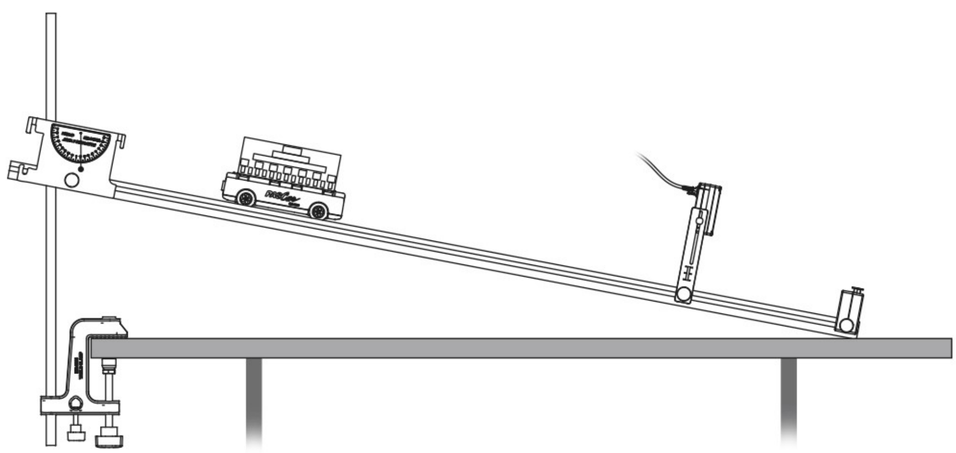


3. Attach the photogate to the photogate bracket. Adjust the height of the photogate on the bracket so the 2.5-cm band on the picket fence passes through the photogate beam as the cart rolls through the photogate.

4. Remove the cart and attach the side of the dynamics track without the end stop to the support rod using the dynamics track rod clamp. Incline the track 10°.

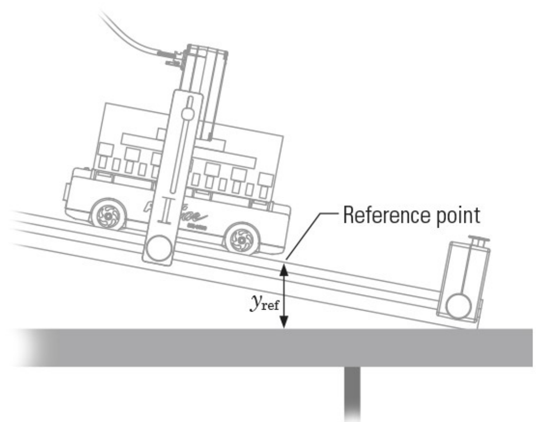
5. Connect the photogate to the data collection system.

6. Configure the data collection system to use photogate timing to measure the speed (or velocity) of the cart as the picket fence passes through the photogate, and then display this measurement in a digits display.



Collect Data

7. Use the balance to measure the mass of the cart and picket fence. Record this mass at the top of the Data Analysis section below.



8. Place the cart on the track and hold it in place with the center of the picket fence aligned with the photogate as in the picture. Make note of the position of the cart's front edge; this will be the reference point from which you will move the cart up the track.

   NOTE: It may be helpful to place a small piece of tape or make a small mark on the track noting the position of the cart's front edge (the reference point).

9. Use the meter stick to measure the height y_{ref} of the cart at the reference point. This height should be measured from the top of the lab table to the top of the track at the front edge of the cart, as shown. Record this value at the top of the Data Analysis section below.

10. Slide the cart up the track 20 cm from the reference point and hold it in place.

11. Use the meter stick to measure the new height of the cart (from the top of the lab table to the top of the track at the front edge of the cart). Record this height in Table 1 next to Trial 1.

12. Begin recording data on the data collection system, and then release the cart, letting it roll freely down the track and through the photogate.

13. Catch the cart just before it hits the end stop, and then stop recording data.

14. Record the speed of the cart at the reference point into Table 1 next to Trial 1 in the Data Analysis section.

15. Repeat the data collection steps four more times, releasing the cart 10 cm farther up the track in each trial. Record the initial height and the speed for each trial into Table 1 in the Data Analysis section.

Data Analysis

Mass of the cart and picket fence (kg): ________________________

Height of the cart at the reference point (m): ________________________

GPE of the cart at the reference point (J): ________________________

1. Use the mass of the cart m, earth's gravitational acceleration $g = 9.8\ \text{m/s}^2$, the height of the cart at the reference point, and Equation 2 to calculate the gravitational potential energy $U_{g\ ref}$ of the cart at the reference point. This is the GPE of the cart at the reference point for each trial. Record the result above.

Table 1: Mechanical energy of a cart on an inclined track

Trial	Initial Height (m)	Speed at Reference Point (m/s)	Initial Gravitational Potential Energy (J)	Kinetic Energy at Reference Point (J)	Total Mechanical Energy at Reference Point (J)
1					
2					
3					
4					
5					

2. Using the cart's initial height from Table 1, calculate the initial gravitational potential energy of the cart for each trial. Record your results in Table 1.

3. Assume that the cart's total mechanical energy when it was released was equal to its initial gravitational potential energy in each trial. Why can you make this assumption?

4. Use the mass m of the cart, the cart's speed v_{ref} at the reference point, and Equation 3 to calculate the cart's kinetic energy K_{ref} at the reference point. Record your results in Table 1.

5. Calculate the cart's total mechanical energy E_{ref} at the reference point by summing its kinetic energy and gravitational potential energy at the reference point. Record your results into Table 1.

6. How does the cart's mechanical energy when it was released compare to its mechanical energy at the reference point?

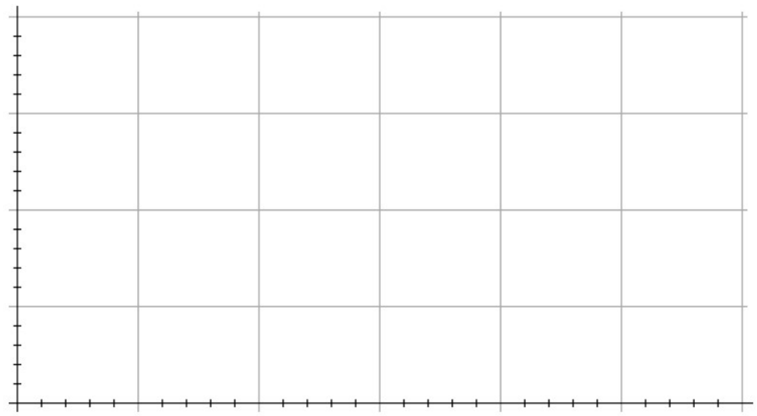
Analysis Questions

1. In your experiment, how did your cart's gravitational potential energy and kinetic energy change as it rolled down the track in each trial?

2. How did your cart's total mechanical energy change as it rolled down the track?

3. Does your data show that the cart's total mechanical energy was conserved? Justify your answer.

4. If you were to plot a graph of your cart's *gravitational potential energy* versus its *kinetic energy* as it rolled down the track, what would be the shape of the curve? Sketch the curve in the blank graph below.

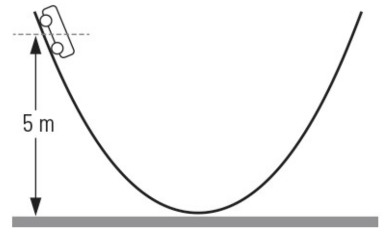


Synthesis Questions

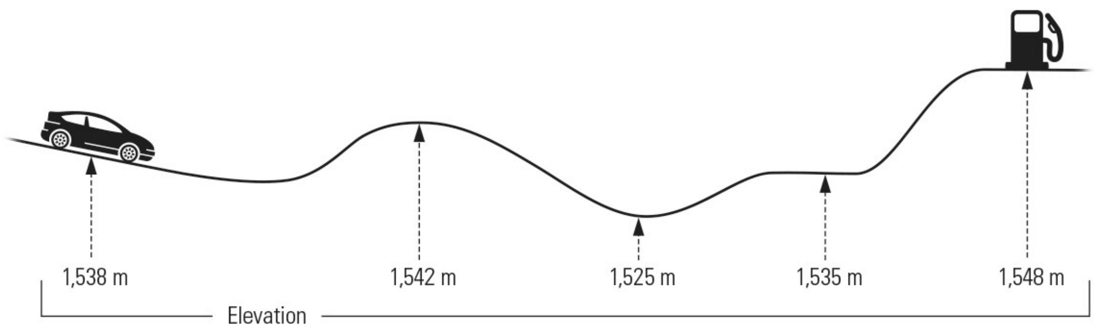
1. Describe how your sketch from the previous question would be different if the cart started with twice as much potential energy.

2. What would be the speed of a cart at the bottom of a very long frictionless inclined plane if it was released from rest at a height of 10.0 m above the bottom of the track? Assume the bottom of the track has height $y_{bottom} = 0$. Show your work.

3. A cart is released from a height of 5 m on the left side of a U-shaped track with identical inclines on the left and right. If the cart is released from rest, how far up (height) the right side of the track will the cart travel? Assume that friction is negligible. Justify your answer.



4. The car shown in the picture below (mass = 998 kg) has just run out of gas while moving at a speed of 15 m/s. Assuming that friction is negligible, will the car make it to the gas station if it coasts the whole way? Justify your answer.



5. In the figure below, a cart is released from rest and rolls down an inclined track and around a loop with radius r = 25.0 cm. From what minimum height y_i must the cart be released so that it does not fall off the track when it is upside down at the top of the loop? Show your work.