17. Simple Harmonic Motion

Grandfather Clock

Objectives

This activity introduces students to the simple harmonic motion of a simple pendulum.

Students will:

- Use a motion sensor to measure the period of a simple pendulum
- Describe the energy conversions taking place during the pendulum's swing
- Gain skills using scientific measurement tools
- Conduct a scientific investigation including making and analyzing graphs, making predictions, and analyzing results

Procedural Overview

Students gain experience conducting the following procedures:

- Setting up the equipment and work area to measure the motion of a swinging pendulum
- Making a simple pendulum from a soda bottle containing a volume of water as its mass
- Measuring the period of the pendulum as it swings through a small angle and a larger angle, first at one mass and then at twice the mass
- Using math skills to interpret the graphs of position versus time

Time Requirement

Introductory discussion and lab activity, Part 1 – Making a simple pendulum	25minutes
Lab activity, Part 2 – Making predictions and Part 3 – Grandfather clock	30 minutes
Analysis	30 minutes

Materials and Equipment

For teacher demonstration:

- ☐ 2-liter soda bottle with cap
- ☐ String, non-stretch, ~2 m
- ☐ Food coloring (optional)
- ☐ Tape

For each student or group:

- ☐ Data collection system
- ☐ Motion sensor
- ☐ 2-liter soda bottle with cap
- ☐ Meter stick
- ☐ String, non-stretch, ~2 m
- ☐ Food dye (optional)
- ☐ Tape
- ☐ Funnel
- ☐ Container of tap water (~500 mL)

Concepts Students Should Already Know

Students should be familiar with the following concepts:

- Different forces
- Potential and kinetic energy
- Energy conversions
- The basics of using the data collection system
- Familiarity with the motion sensor

Related Labs in This Guide

Labs conceptually related to this one include:

- Investigating Seismic Waves

Using Your Data Collection System

Students use the following technical procedures in this activity. The instructions for them are in the appendix that corresponds to your PASCO data collection system (identified by the number following the symbol: "�"). Please make copies of these instructions available for your students.

- Starting a new experiment on the data collection system �(1.2)
- Connecting a sensor to the data collection system �(2.1)
- Recording a run of data �(6.2)
- Displaying data in a graph �(7.1.1)
- Adjusting the scale of a graph �(7.1.2)
- Saving your experiment �(11.1)

Background

Potential energy is stored energy due to an object's position or to the energy associated with chemical bonds. There are different types of potential energy. Gravitational potential energy is due to an object's height above ground. Elastic potential energy can be found in objects such as a stretched rubber band. Elastic materials, such as a spring, trampoline, or rubber band, resist being stretched out of shape.

The different types of potential energy can be converted to kinetic energy. For example, toy airplanes fly as a twisted rubber band unwinds and spins a propeller. The elastic potential energy in the rubber band was converted into kinetic energy. Kinetic energy is the energy of motion. The sum of the potential and kinetic energies make up an object's mechanical energy. The weight of an object is the measure of gravity acting on an object.

A simple pendulum consists of a point mass (a "bob") suspended from a string of negligible mass. Such a system exhibits periodic motion in which it vibrates at a resonant frequency. An example of this type of system is a child's swing on the playground. It is easy to start a swing in motion with a little push. But just by giving the swing the same little push at the right point, you can cause it to go higher and higher. This is because the swing has a resonant frequency, or a frequency at which it "wants" to swing.

Periodic motion is motion that repeats itself in a precise pattern. The simple harmonic motion (SHM) of a pendulum is periodic motion that we describe by its period, frequency, and amplitude. The period is the time it takes the pendulum to complete one full swing, from a starting point back to that same point. The pendulum, therefore, passes through this point twice during each period. The frequency is the number of complete swings made each second. Frequency is usually referred to in terms of cycles per second. The amplitude is the maximum displacement of the pendulum from its equilibrium point, or where it hangs when it is motionless. For small amplitudes of swing (those of a few degrees), the period of a simple pendulum is approximated by the following equation:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where T is the period in seconds, L is the length of the string in meters, and g is the acceleration due to the gravitational force, and has an approximate value of 9.8 m/s^2.

As the pendulum swings back and forth through its arc, its kinetic energy is converted to potential energy and vice versa. Its kinetic energy is at a maximum as it passes through its equilibrium point; its potential energy is at a maximum when it is at its maximum displacement, or distance from its equilibrium point.

Although pendulums have been in existence for many years, it was Galileo who analyzed and codified the functioning of pendulums in the late 16th century. As he was watching a chandelier swing back and forth in the cathedral, he wondered if the length of the chandelier's chain had anything to do with how long it took the chandelier to complete one swing and if a bigger swing took longer to complete than a smaller one. After exhaustive and painstaking experimental work with a variety of pendulums in his laboratory, Galileo determined that:

- Pendulums will always return to almost exactly the same height from which they were released.
- Pendulums eventually come to rest at their equilibrium point, but it takes heavier ones longer to do this than do lighter ones.
- The period of a pendulum is not dependent on the mass of its bob.

- The period of a pendulum is not dependent on the amplitude of its swing.
- The square of the period varies directly with the length of the pendulum.

Galileo's work with pendulums led the way to accurate clocks, as well as much of the basis for the study of motion in modern physics. Today's grandfather clocks are the descendants of Galileo's pendulum timekeepers. The central feature of the grandfather clock is its size, particularly its height. If you look closely at a grandfather clock, you can see that most of its height is due to the pendulum it uses to mark time.

Pre-Lab Discussion and Activity

Engage students in the following discussion or activity:

Ask the students to name and describe the device that musicians use to keep time when they practice a piece of music. Some students who are learning to play the piano may even have these at home.

> Musicians use a metronome. The mechanical models use an inverted pendulum that swings back and forth on an upright metal arm. The pendulum has a mass that can be slid up and down the arm to adjust its tempo or beat. The farther up the arm the mass is, the slower the beat, while the lower down the arm, the faster the beat. Electronic metronomes usually use the vibration of a quartz crystal instead of a metal arm with a mass.

Ask students which piece of playground equipment works most like the metronome. Is the motion of this toy random or is there a pattern? Describe the playground toy's energy conversions as it goes through its motion.

> A swing at the playground works much like a metronome because they are both a type of pendulum. The motion is not random, but has a repeating pattern of back-and-forth. Gravitational potential energy is converted into kinetic energy and back; a person on a swing speeds up during the first half of the swing's arc, then starts to slow down as the swing reaches the highest point of the arc on the opposite side until stopping and reversing direction. The person speeds up until passing through the bottom of the arc and then momentarily stopping again at the top of the arc.

Direct the students to “Thinking About the Question.” Discuss the answers to these questions as a whole group.

Ask students if they think a soda bottle pendulum will behave like a playground swing. Specifically, do they think the pendulum will return to its initial height, *but no higher*, when it is allowed to drop? Show the students the demonstration you have rehearsed ahead of time, using yourself or a volunteer to drop the soda bottle.

Now direct the students to “Investigating the Question.”

Preparation and Tips

Set up these materials and equipment prior to the lab:

- One day before the lab:
    - Remove the label and fill a 2-liter soda bottle about halfway with water. Add some food dye to color the water so students can clearly see the volume of water. Cap the bottle. Suspend the bottle from the ceiling of the classroom to allow for the longest possible length of string. Have this demonstration set up for students to see at the beginning of the pre-lab discussion.

- Before involving the students in the pre-lab activity, *rehearse ahead of time* the following demonstration until you can perform it confidently:

  1. Pull the soda bottle pendulum back to a height where it just touches your chin or nose.
  2. Carefully, without pushing the pendulum, release it from your hands and allow it to fall.
  3. Do not move.
  4. Allow the pendulum to complete one swing so that it comes almost exactly back to your chin or nose.
  5. On the second swing, grab hold of the soda bottle to bring the pendulum to rest, and return it to its equilibrium position (hanging straight down).

- On the day of the lab, you can ask for student volunteers to perform the demonstration or you can perform it yourself. If you use a student volunteer, consider standing just behind him or her and dropping the bottle yourself to ensure that it is not pushed. To heighten the suspense, position yourself (or the volunteer) with your back to a wall so it is not possible to back away from the pendulum on its return part of the swing.

- Provide the maximum height possible in your classroom for students to suspend their pendulums. The longer the strings they are able to use, the greater the variation they can discover in the periods of their pendulums. Ensure that the support to which the students are attaching their pendulums is rigid; if pendulums are attached to something moveable they will transfer some of their energy to it.

- Show students how to shorten the pendulum's string without disassembling it by tying a loop in the string.

Safety

Add this important safety precaution to your normal laboratory procedures:

- Ensure that students have enough room to safely swing their pendulums.

Driving Question

How does a pendulum measure time?

Thinking about the Question

If you have ever been in a swing, you know that you can be pushed by a friend or family member or you can move in such a way that you start the swing going on your own. If you start the swing on your own, you can make it go higher and higher by adding your own energy to it at just the right moment in the swing. If you are good at this, you can get the swing going to an exciting height. What happens when it is time to slow down and stop the swing? In your lab group, discuss what happens when you stop adding energy to the swing and just sit in it while it slows down. Be prepared to share your thoughts with the class.

> Students should suggest that when they stop adding energy to the swing it immediately begins slowing down. Each swing, or arc, is not quite as high above the ground as the preceding one and the swing eventually comes to a stop after a series of diminishing arcs.

What is gravitational potential energy?

> Gravitational potential energy is stored energy due to an object's position.

What is kinetic energy?

> Kinetic energy is the energy associated with a moving object.

Watch your teacher's demonstration involving the soda bottle pendulum.

Sequencing Challenge

Note: This is an optional ancillary activity.

The steps below are part of the Procedure for this lab activity. They are not in the right order. Determine the proper order and write numbers in the circles that put the steps in the correct sequence.

4	3	5	1	2
Suspend the soda bottle from an overhead support, positioning the bottle 20 cm above the motion sensor's screen.	Tie a long string to the neck of the soda bottle and make a secure knot.	Record position data as you allow the soda bottle to make a series of swings back and forth.	Make certain that each lab group member is aware of the safety rules and procedures for this lab.	Fill a 2-liter plastic soda bottle one-quarter full of water and replace the lid.

Investigating the Question

Note: When students see the symbol "�" with a superscripted number following a step, they should refer to the numbered Tech Tips listed in the Tech Tips appendix that corresponds to your PASCO data collection system. There they will find detailed technical instructions for performing that step. Please make copies of these instructions available for your students.

Part I – Making a simple pendulum

1. ☐ Fill an empty 2-liter plastic soda bottle one-quarter full with water.

2. ☐ Add food coloring to the water in the soda bottle.

3. ☐ Cap the bottle tightly. This part of the pendulum system is referred to as the "bob."

4. ☐ Tie one end of a long piece of string around the neck of the soda bottle, under the ring that protrudes from the neck. Make sure the string is securely tied with a knot.

5. ☐ Tie the other end of the string to the ceiling or to the support provided by your teacher.

Part 2 – Making predictions

6. ☐ Write your predictions for the following:

   a. How many seconds do you think it will take your pendulum to make one complete swing from one side to the other and back again?

   Answers will vary. Students may suggest times ranging from a fraction of a second to several seconds.

   b. Will it take more time, less time, or the same amount of time to make one complete swing if you pull the pendulum back farther before letting it go?

   Answers will vary. Students' intuition may lead them to suggest that it will take more time the farther back they pull the pendulum, but this is incorrect; the time will be the same regardless of the release point.

   c. How will the amount of time to make one complete swing be affected if you double the amount of water in your soda bottle?

   Answers will vary. Students' intuition may lead them to suggest that it will take more time the heavier they make the pendulum, but this is incorrect; the time will be the same regardless of the mass of water.

   d. How will the amount of time to make one complete swing be affected if you change the length of the pendulum's string by shortening or lengthening it?

   Answers will vary. The time will decrease for shorter lengths of string, and increase for longer lengths of string.

Part 3 – Grandfather clock

7. ☐ Start a new experiment on the data collection system. �(1.2)

8. ☐ Connect the motion sensor to the data collection system. �(2.1)

9. ☐ Display Position on the y-axis of a graph with Time on the x-axis. �(7.1.1)

10. ☐ Adjust the height of the soda bottle pendulum so that when the motion sensor is placed on the floor beneath it, there is about 20 cm of space between the bottom of the bottle and the metal screen of the motion sensor.

    Note: The motion sensor should be set to the "person" icon.

11. ☐ Begin data recording. �(6.2)

12. ☐ Gently pull the soda bottle back a few centimeters, so it is just beyond the motion sensor, and let it go. Try not to push it or add any extra vibrations to the pendulum. Why is it important to let the pendulum fall on its own without pushing it?

It is important not to push the pendulum bob because that gives it extra kinetic energy to begin with. At the beginning it should only have gravitational potential energy due to its position.

13. ☐ Allow the pendulum to swing for 20 seconds.

14. ☐ Stop data recording. �(6.2)

15. ☐ Examine your data of position versus time. You may need to adjust the scale of your graph. �(7.1.2) How can you tell from the graph the number of swings the pendulum made in a certain amount of time?

We can tell how many swings the pendulum made by counting the number of spikes in the graph because each spike represents the moment when the soda bottle crossed over the motion sensor.

16. ☐ Begin data recording. �(6.2) This will be your second data run on the same graph.

17. ☐ Gently pull the soda bottle back, this time about twice as far beyond the motion sensor, and let it go.

18. ☐ Allow the pendulum to swing for 20 seconds.

19. ☐ Stop data recording. �(6.2) Based on your data, what do you notice about the number of swings the pendulum made in the second trial compared to the first trial?

Students should observe that the number of swings per period of time remains the same in the two trials.

20. ☐ Carefully remove the cap of the soda bottle without disconnecting the pendulum from the system.

21. ☐ Pour in enough water to double the volume. By doubling the volume of the water, what other property of the water do you double?

Doubling the volume also doubles the mass. For example, 500 mL of water has a mass of 500 g; 1000 mL of water has a mass of 1000 g.

22. ☐ Begin data recording. �(6.2) This will be your third data run on the same graph.

23. ☐ Gently pull the soda bottle back a few centimeters, so it is just beyond the motion sensor, and let it go.

24. ☐ Allow the pendulum to swing for 20 seconds.

25. ☐ Stop data recording. �(6.2) Based on your data, what do you notice about the number of swings the pendulum made in this trial compared to the previous trials?

Students should observe that the number of swings per period of time remains the same in all the trials.

26. ☐ Begin data recording. �(6.2) This will be your fourth data run on the same graph.

27. ☐ Gently pull the soda bottle back, this time about twice as far beyond the motion sensor, and let it go.

28. ☐ Allow the pendulum to swing for 20 seconds.

29. ☐ Stop data recording. �(6.2) What effect, if any, does the distance from the motion sensor have on the number of swings the pendulum makes per period of time?

Students should observe that there is no significant change in the number of swings per period of time.

30. ☐ Shorten the string of your pendulum so it is about half its initial length. One way to do this is to tie a loop into the string.

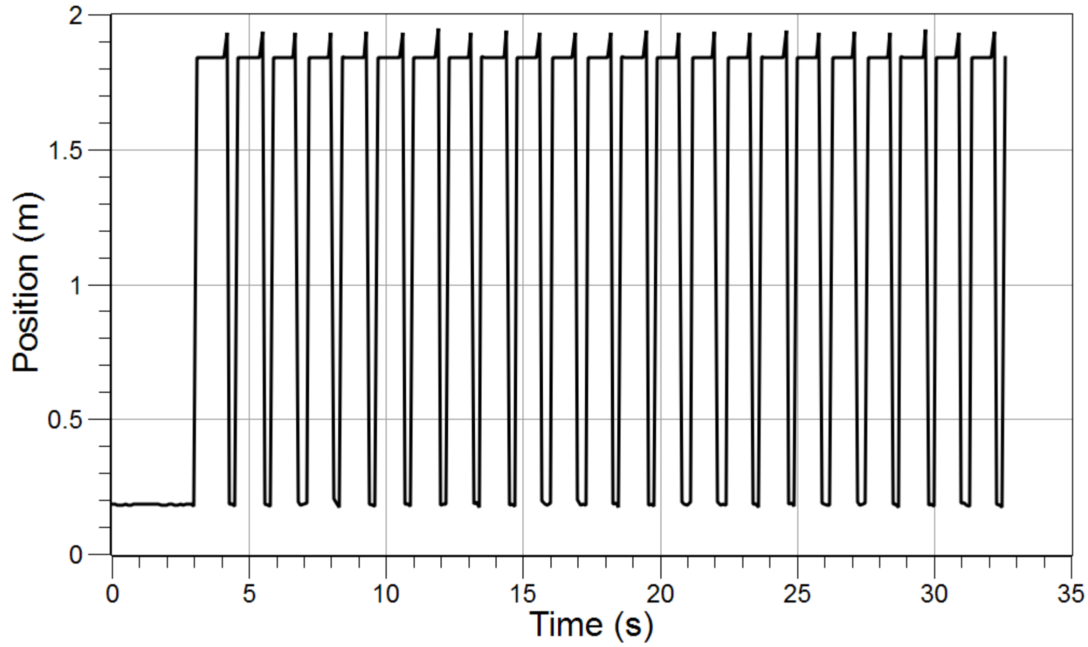
31. ☐ Begin data recording. �(6.2) This will be your fifth data run on the same graph.

32. ☐ Allow the pendulum to swing for 20 seconds.

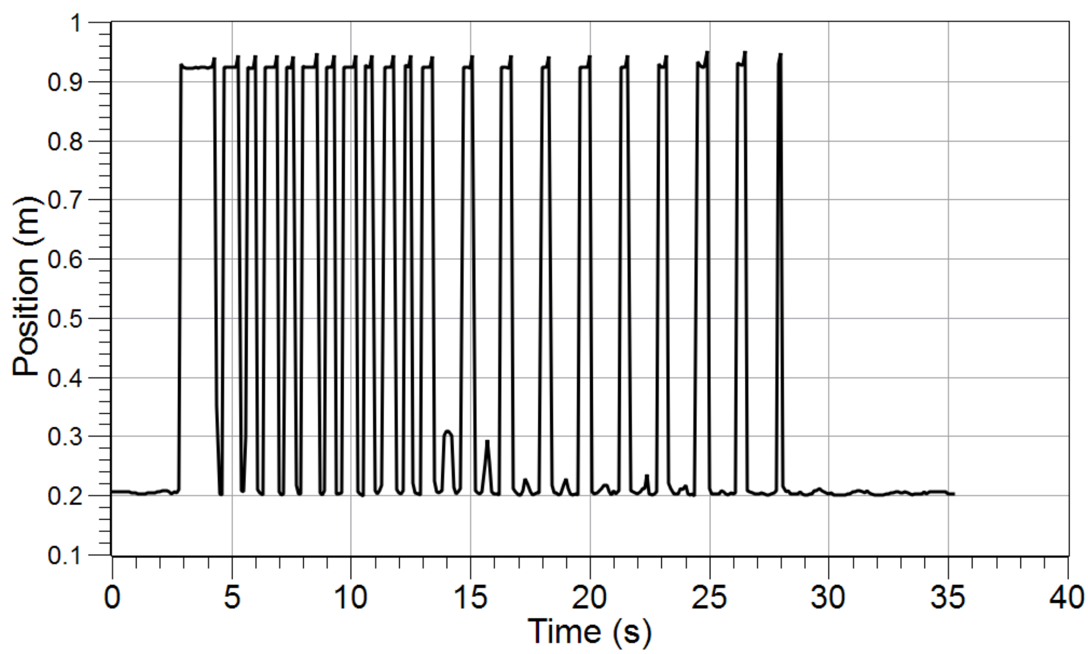
33. ☐ Stop data recording. �(6.2)

34. ☐ Save your experiment according to your teacher's instructions. �(11.1)

Sample Data



Pendulum with longer string



Pendulum with shorter string

Answering the Question

Analysis

1. How did your predictions from Part 2 compare to your experimental results?

   Answers will vary. One group answered as follows: We predicted that the amount of time would be longer for our pendulum to swing when we pulled it back farther and when it was twice as heavy with water. We thought it would take less time to make each swing when the string was shortened. Our results showed that the time changed only when we changed the length of the string. When we shortened it, it did take less time to make its swings.

2. Review your data carefully. How many times must the soda bottle bob pass across the motion sensor as it travels through one complete cycle? Explain why this is the case.

   The bottle has to pass across the motion sensor twice to make one complete cycle. This is because a cycle is defined as the pendulum beginning its motion at a particular point and returning to that same point again, like a round trip. If we just considered once past the motion sensor, this would be like making a one-way trip.

3. Draw the pendulum system, including a representation of the pendulum's swing, and label and describe the energy conversions that are taking place. Be prepared to share your drawing and explanations with the class.

   At the top of its swing, all of the energy the pendulum has is gravitational potential energy (GPE). The force of gravity is what is causing the pendulum to fall. On the way down, the pendulum's GPE is being converted into kinetic energy (KE) and the pendulum is accelerating. At this lowest point of the arc, all of the energy is KE that is available in the system. Some of the energy in the system has been converted into thermal energy, due to friction between the string and the support.

4. How much time is required for the pendulum, at the initial length of string, to make one complete cycle?

   Answers will vary. One student group found that it took 2.5 seconds for the pendulum at its initial length to make one complete cycle.

5. How much time is required for the pendulum, at the shortened length of string, to make one complete cycle?

   Answers will vary. One student group found that it took 1.7 seconds for the pendulum at its shortened length to make one complete cycle.

6. The behavior of a simple pendulum, such as the one you made and used in this activity, is ideal for use as a time-keeper. In fact, the metronome used by musicians is an example of a type of time-keeping pendulum. Another example is a grandfather clock. The behaviors which are most helpful for time-keeping are listed below:

   1. The period of a pendulum does not depend on the mass of its bob.
   2. The period of a pendulum does not depend on the size (amplitude) of its swing.
   3. The square of the period varies directly with the length of a pendulum.

a. What evidence does your data provide that supports the first two behaviors?

Answers will vary. One group answered as follows: When we analyze our data we can see that the time it took for the pendulum to make a complete swing was the same when it was both 1/4 full of water and 1/2 full of water. Since the amount of water changed the mass, this shows evidence that changing the mass does not affect the period or time to make one complete swing. Also, our data shows that whether we pull the pendulum bob back just a little distance or a larger distance, the time for one complete swing is not changed. This is evidence that the size or amplitude of the swing does not affect the time to make one complete cycle.

b. What further information would you need to know about your pendulum system to be able to discuss the third behavior?

We would need to know the length of the string of our pendulum.

7. A lab group has set up a pendulum system, has conducted multiple trials to measure the time of the swings, and has measured the length of the pendulum, finding it to be 72 cm long. They have done some research on pendulums and discovered an equation that relates the length of the pendulum to its period. The equation is:

$$Period = 2\pi\sqrt{\frac{\text{Length of pendulum (in meters)}}{g}}$$

The students found the following information in their science text: The variable g stands for the acceleration due to gravity, and has an approximate value of 9.8 m/s^2.

How much time, on average, does it take this lab group's pendulum to complete each cycle of back-and-forth swing? Show your work. Remember to convert the length measurement to meters.

$$\text{Period} = 2\pi\sqrt{\frac{\text{Length of pendulum (in meters)}}{g}}$$ Length Conversion: 72 cm = 0.72 m

$$\text{Period} = 2\pi\sqrt{\frac{0.72\text{ m}}{9.8\text{m / s}^2}}$$

$$\text{Period} = 2\pi\sqrt{0.0735}$$

$$\text{Period} = 2\pi(0.271)$$

$$\text{Period} = 1.7\text{s}$$

8. Pendulums will always return to almost exactly the same height from which they were released. What evidence do you have from this lab activity that supports this statement about pendulums?

Answers will vary. Students should reference the pre-lab demonstration in which the pendulum returned almost to touch the tip of the demonstrator's chin. They may also suggest that their pendulums returned to nearly their initial starting heights.

Multiple Choice

Circle the best answer or completion to each of the questions or incomplete statements below.

1. Motion that repeats itself in a predictable pattern is called:

   A. Simple harmonic motion
   B. Newtonian motion
   C. Chaotic motion

2. The size of a pendulum's swing is also known as the pendulum's:

   A. Amplitude
   B. Period
   C. Cycle

3. Each complete back-and-forth swing a pendulum makes is referred to as a/an:

   A. Amplitude
   B. Period
   C. Cycle

4. The amount of time required for a pendulum to complete one back-and-forth swing is:

   A. The pendulum's amplitude
   B. The pendulum's period
   C. The pendulum's cycle

5. Because of the simple harmonic motion of pendulums, they are useful for:

   A. Maintaining lengths of string under tension
   B. Measuring the passage of time
   C. Demonstrating random motion

6. At the top of its swing, or the maximum distance from equilibrium, a pendulum's energy is:

   A. Entirely kinetic energy
   B. Almost equally kinetic energy and gravitational potential energy
   C. Entirely gravitational potential energy

7. A grandfather clock's pendulum has a small adjustment knob on its bob so that in case it is running fast or slow, it can be adjusted to keep better time. What must this knob adjust in order to change the clock so it keeps accurate time?

   A. The mass of the pendulum (the knob adds or removes mass).
   B. The distance the knob can swing (the knob changes the amplitude)
   C. The length of the pendulum (the knob lengthens or shortens the pendulum)

8. In order for a pendulum to continue swinging, what must be added to it?

   A. Energy
   B. Mass
   C. Time

9. An object that is not being subjected to a force will continue to move at a constant speed, yet the pendulum eventually stops swinging and comes to rest in its equilibrium position. What force or forces cause this simple harmonic motion to stop?

A. Gravity and friction

B. Gravity and magnetism

C. Magnetism and friction

10. Which of the following statements about energy is **not** true?

A. Energy can neither be created nor destroyed; it can only be transformed from one kind to another.

B. Gravitational potential energy is stored energy due to an object's position and height above the ground.

C. Kinetic energy is the one form of energy that is not conserved; it can be both created and destroyed.

Further Investigations

Do research to find out where some of the largest pendulums are located and what they look like. How long are the periods of some of these pendulums? In your research, try to discover why some of these large pendulums are displayed with a circular border around them on which the hours of the day are marked. Does the pendulum rotate or does the earth rotate under the pendulum?

Investigate how a pendulum can be used as a seismic wave detector.

Design an experiment to see how much you can vary the period of a pendulum. Can you make a pendulum whose period exceeds 3 seconds? Even longer?

Is there such a thing as a chaotic pendulum or is this a contradiction in terms?

What are damped and driven pendulums?

Rubric

For scoring students' accomplishments and performance in the different sections of this laboratory activity, refer to the Activity Rubric in the Introduction.